The Real Options Effects of Uncertainty on Investment and Labour Demand

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Abstract

This thesis considers how uncertainty affects firms' investment and employment behaviour. Chapter 1 summarizes the thesis. Chapter 2 shows that, contrary to common beliefs, the real options effect of uncertainty plays no role in the long run rate of investment. Real options and irreversibility, however, are shown to play an important role in the short run dynamics of investment and labor demand. Specifically, they reduce the short run response of investment and hiring to current demand shocks, and lead to a lagged response to past demand shocks.

Chapter 3 derives robust empirical predictions on the effects of uncertainty on investment dynamics. This approach is based on an explicit aggregation across plants and lines of capital to the firm level, and is compatible with a general class of production functions and demand processes. An investment model is estimated on a panel of UK firms and finds that, as predicted, uncertainty significantly reduces firms' responsiveness to demand shocks.

Chapters 4 extends these results by using a structural estimator of investment and labour demand at the plant level. Firms are modelled as operating a number of these separate production plants, which are subject to simulated plant level shocks and an observable firm level shock. Indirect inference is used to estimate the free parameters, and successfully match the time se-
ries and cross sectional behaviour of investment and hiring in a panel of US Compustat firms.

Chapter 5 uses real options theory to model the investment in embodying newly patented products and processes. Patent data is collected for a panel of UK firms, and is shown to raises firms’ market values and productivity. The effect on productivity, however, is reduced by higher levels of uncertainty, in line with predictions from the real options framework that uncertainty reduces the rate of embodiment of new-technologies.
Acknowledgements

I would particularly like to thank John Van Reenen, Steve Bond, Richard Blundell, Rachel Griffith and Frank Windmeijer for their help. Chapter 2 is joint work with Steve Bond and John Van Reenen, and Chapter 4 is joint work with John Van Reenen. Part finance for this thesis has been provided by the E.S.R.C. Centre for the Microeconometric Analysis of Fiscal Policy at I.F.S. and is gratefully acknowledged.
Declaration

No part of this thesis has previously been presented to any University for any degree.

Nicholas Bloom
Chapter 1

Introduction and Summary of the Thesis

This thesis sets out to clarify and test the implications of real options on firms’ investment and employment decisions. Real options describe the value a firm will put on being able to ‘wait and see’ how market conditions evolve before committing to investment and hiring decisions. These real options will play a role whenever the following three conditions are met:

1. Investment or employment decisions involve some sunk costs - that is they are partially irreversible.

2. Future business conditions are uncertain, but with new information arriving over time. For example, demand or costs may follow an autoregressive process.

3. The firm has the option to delay its decision making.

In these circumstances the firm will be tempted to wait and evaluate future market conditions before committing partially irreversible decisions. Since many investment and hiring decisions will satisfy these conditions real
options can be expected to play an important decision making role. Furthermore, as Pindyck (1988) has shown real options can account for more than half a firm's value, so that their effects can be quantitatively important.

In chapter 2 of the thesis I synthesize and extend the prevailing theoretical literature on real options. Contradictory to the implications of Caballero (1991), Pindyck (1993), Sakellaris (1993), Dixit and Pindyck (1994), and Shin and Lee (2000) I prove that real options, under standard homogeneity conditions, will have no effect on the long run rate of investment. This is shown both for the standard Cobb-Douglas production and Geometric Brownian motion demand model examined in Abel and Eberly (1996), and a more general class of models examined in Eberly and Van Miegem (1997).

This result is important because most prior empirical work has attempted to test the real options literature by examining its direct effect on the rate of investment. This chapter implies this is inappropriate since there is no direct long run real options effect. I then show that while real options play no long run role they will exert a powerful short run influence on firms' investment and hiring behaviour. In particular the presence of real option values will reduce a firm's response to demand shocks. In essence, real options will make firms more cautious when making investment or employment decisions in uncertain market conditions.

In chapter 3 of the thesis, which is a joint with Steve Bond and John Van Reenen, we set out to empirically test the predictions of the real options framework. To do this we have to deal with two obstacles which have impeded much of the previous empirical work. The first is aggregation, which obscures the predictions of the real options models in firm, industry or macro-level data sets. The threshold investment rules implied by the real options models predict lumpy and frequently zero investment. While this is observed in some
micro plant-level data such as the US LRD or the UK ARD, at the firm level investment and hiring is much smoother and rarely zero. Therefore, we develop Eberly and Van Miegem’s (1997) model to generate an empirical framework for dealing with aggregation, which provides predictions which are robust to arbitrary aggregation across multiple production plants and lines of capital within firms.

The second obstacle requires modelling the effects of time varying uncertainty on real options in our general estimation framework. Prior theoretical work has commonly assumed time constant uncertainty within each firm. Since we use a time varying empirical uncertainty measure we allow for time varying uncertainty in our theoretical framework, and show that as predicted, higher uncertainty defined in terms of second order stochastic dominance leads to larger short run real options effects.

The predictions of our theoretical framework are then empirically tested on a panel of UK firm level data using the GMM system estimation framework suggested by Blundell and Bond (1998). The results suggest that uncertainty plays an important role in retarding the response of investment to demand - that is higher uncertainty makes firms more cautious and leads them to respond less strongly to changing demand conditions. For policy makers this implies that higher micro and macro economic uncertainty will reduce firms’ investment and employment responses to policy interventions, such as tax and interest rate changes.

One exception is Hassler (1996) who examines the effects of time varying uncertainty, but in a fixed cost structural framework. While these results are instructive they cannot be extended to our general framework.

Interestingly, earlier work on investment under uncertainty by Nickell (1977), which concentrated on the interaction of delivery lags and uncertainty, came to a similar conclusion that higher uncertainty will lead to a more gradual and cautious investment response. Empirically identifying this framework from the real options approach is the subject of a future research project.
Chapter 4 also attempts to empirically model the effects of real options on investment and labour demand, but from a much more structural framework than Chapter 3. By making stronger assumptions on the nature of firm level production and demand shocks I am able to make much stronger structural predictions. I use Eberly and Van Miegem’s (1997) results to structurally characterise firm’s optimal joint investment and employment behaviour. This is undertaken in a two stage modelling process. First, I model the firm as operating a number of separate production projects. Each project is assumed to operate with a Cobb-Douglas capital and labour production function and be subject to Brownian motion demand shocks. This allows me to explicitly solve their optimal investment and hiring strategies.

Second, these projects are assumed to face a common firm level shock taken from the firm level sales data and an idiosyncratic project level shock. This project level shock is simulated and the project level investment and hiring responses aggregated up to the firm level.

Using this simulation approach I can then develop a structural estimator of joint firm level hiring and investment decisions. This is estimated using the indirect inference approach developed by Gourieroux, Monfort and Renault (1993) to uncover the free structural parameters. Based on these parameters the predicted investment and employment responses are shown to closely mimic the cross sectional and time series behaviour of actual investment and employment in my Compustat panel of US firms. The structural nature of this approach would allow me to use it to forecast the effects of policy interventions, such as interest rate or tax changes, on investment and employment.

Chapter 5 of thesis, which is co-authored with John Van Reenen, looks at patent data to investigate the effects of real options on a different kind of in-
vestment, the uptake of new technologies. Analysing the new IFS-Leverhulme database on over 200 major British firms since 1968 we show that patents have a statistically significant impact on firm-level productivity and market value. But while patenting feeds immediately into market values it has a slower impact on productivity. This appears to be because of the need for costly investment in new equipment, training and marketing required to embody patents into new products and processes. Since patents provide firms with the option to delay investment without threat from competitors, these sunk costs of embodiment will generate valuable real options when market conditions are uncertain.

This is supported by our empirical results which find that higher market uncertainty, which increases the value of real options, reduces the impact of new patents on productivity. This suggests that delivering micro and macro and micro stability could be an important channel for policy makers to encourage the take up of new technologies and productivity growth.
Chapter 2

The Real Options Effect of Uncertainty on Investment and Labour Demand

2.1 Abstract

This paper shows that, contrary to common beliefs, the real options effect of uncertainty plays no role in the long run rate of investment. This is proven for both the standard investment model with Cobb-Douglas production and Brownian motion demand, and also for a broader class of models with multiple lines of capital, labour and general demand stochastics. Real options and irreversibility, however, are shown to play an important role in the short run dynamics of investment and labour demand. Specifically, they reduce the short run response of investment and hiring to current demand shocks, and lead to a lagged response to past demand shocks.
2.2 Introduction

It is commonly understood that the real options effect of uncertainty reduces investment\(^1\). To be precise, when investment is irreversible so that capital cannot be resold for its full purchase price, a firm's optimal investment rule takes on a threshold form. Investment will only occur when demand rises to some upper threshold while disinvestment will occur only when demand falls to some lower threshold. It has often been assumed that because uncertainty raises the upper threshold for investment\(^1\), it reduces the long run rate of investment.

This appears to be confirmed by a variety of papers which find an inverse relationship between uncertainty and investment. Caballero (1991) and Lee and Shin (2000) demonstrate that in a two period model in which firms start off with no capital stock the real options effect of uncertainty unambiguously acts to reduce investment in the first period. Pindyck (1993) and Sakellaris (1994) report that in a competitive multi-period (three or more periods) model of investment in which firms again start off with no initial capital stock the real options effect of uncertainty also unambiguously reduces first period investment. And Dixit and Pindyck’s (1994) survey on the investment under uncertainty literature implies a negative impact of real options on investment when they report that

"a larger \( \sigma \) [the standard deviation of demand] means a lower
long-run average growth rate of the capital stock, and thus less

\(^1\)The real options effect of uncertainty is defined as "the effect of uncertainty that arises from a firm's option to choose the timing of its investment" when this investment is irreversible.

\(^2\)The effect of uncertainty in raising the investment threshold is demonstrated, for example, by Bertola (1988), Pindyck (1988), Dixit (1989), Bentolila and Bertola (1990), Bertola and Caballero (1994), Dixit and Pindyck (1994) and Abel and Eberly (1996).
investment on average” [page 373]

In this paper we argue that these existing papers cannot be used to draw inferences about the real options effect of uncertainty on long run investment since they either amalgamate differing short and long run impacts, or introduce Jensen’s inequality effects in to demand growth. This paper distinguishes the real options effect in the short run from its effect in the long run, and controls for these Jensen’s inequality effects. In section 2 we show that real options play no role in determining the long run rate of investment.

The intuition for this result is that while the real option effect of uncertainty increases the investment threshold, reducing the rate of investment in times of strong demand, it also lowers the disinvestment threshold, reducing the rate of disinvestment in times of weak demand. These effects on the rate of investment and the rate of disinvestment exactly cancel out in the long run. This result is proven for both the standard Cobb-Douglas production function and Brownian motion demand model (proposition 1) and also a more general class of multi-capital production functions and stochastic demand processes (proposition 2). As a corollary to proposition 1 we also show that the negative effect of uncertainty on long run investment quoted above from Dixit and Pindyck (1994) derives from a Jensen’s inequality effect and is unrelated to real options.

Although real options do not affect long run investment, it is shown in section 3 that they play an important role in the short run dynamics of investment and labour demand. The separation of the investment/disinvestment thresholds and the hiring/firing thresholds is proven to reduce the short run responsiveness to demand shocks at any level of aggregation (proposition

\footnote{An alternative intuition for this result is that log capital is stationary around its perfectly reversible level for any level of uncertainty. Therefore, these are are cointegrated with a common growth rate.}
3). This explains the findings of Caballero (1991), Pindyck (1993), Salkellaris (1994) and Lee and Shin (2000) whose assumption that firms start off with no capital is critical since it ensures that first period investment will necessarily be positive. Because higher uncertainty reduces the investment response it reduces this first period investment in their model and generates a negative real options effect of uncertainty on investment. We show that the investment and hiring response to demand shocks is convex with larger shocks leading to proportionally larger responses (proposition 4). In addition to reducing the short run responsiveness, real options and irreversibilities are also shown to induce strong dynamics into the investment and hiring process, leading to lagged responses to past demand shocks (proposition 5).

These predictions from real options and irreversibility - in particular the low short run responsiveness and the longer lagged response - match the stylized facts from estimating firm, industry and macro level investment and labour demand models\(^4\). Estimates of the short run elasticities of investment and hiring to demand shocks are generally moderate in size, with point estimates on annual data often around one quarter and one half respectively. Additional lagged responses, however, lead to a much higher long run response, with some suggestion that the long run elasticities of both factors is about unity.

Related literature to this paper includes Abel and Eberly (1999) who examine the influence of uncertainty on the level of the capital stock under complete irreversibility with no depreciation for a range of parameter values, and find an ambiguous response. Our work complements these results by examining the real options effect of uncertainty and irreversibility on long

run investment - that is on the growth of the capital stock. Since we find no impact on the long run growth of the capital stock this suggests that any real options effects of uncertainty on the level of the capital stock are stationary and independent of long run investment. Our results are also fully analytical, unambiguous, and obtained for a more general model which allows for partial irreversibility, depreciation, multiple-lines of capital, and broader demand stochastics.

The impact of real options and irreversibilities on short run dynamics has previously been noted by Bertola and Caballero (1994), who model aggregate investment in an economy composed of a continuum of homogenous firms, each operating with a single line of irreversible capital. We extend these results in a number of directions by generalizing to firms with multiple-lines of partially irreversible labour and capital, allowing for a less restrictive demand processes, allowing for heterogeneity across firms, and dispensing with the need for aggregation across a large numbers of units. This allows us to make predictions on the short run dynamic effects of real options on investment and hiring in data-sets at all levels of aggregation, from the plant and firm level up to the industry and macro level, with these being robust to a variety of production and demand processes.

Finally, a number of papers cited above also examine a result, demonstrated in Hartman (1972) and Abel (1983), that if a firm can freely adjust its labour force used in production after investment has been undertaken, this can lead the marginal revenue product of capital to be convex in price, so that greater uncertainty may increase the level of the capital stock. The result, however, depends on particular modelling assumptions over both the revenue function and the form of the demand shock so that, for example, if

\footnote{Caballero (1991), Pindyck (1993), Sakellaris (1994) and Lee and Shin (2000).}
the demand shock is modelled as a quantity rather than a price shock this uncertainty effect disappears or can even be reversed. This Hartman-Abel effect of uncertainty also requires firms to undertake frequent adjustment of its labour force, which if labour is believed to be subject to adjustment costs, may not be optimal. Since our results are valid both with and without this uncertainty effect we do not discuss this issue any further.

The plan of the paper is as follows. Section 2 starts by examining the standard model with a single line of capital and flexible labour and demonstrates that real options have no impact on the long run rate of investment. We then consider a broad class of models which allow for multiple lines of capital and inflexible labour under any degree of aggregation, and show that real options still play no role in determining the long run rate of investment. Section 3 then uses this general framework to consider the short run effects of real options and irreversibilities, and proves that these will retard the investment and hiring response to demand shocks, and lead to a dynamic lagged response to demand shocks. Some concluding remarks are then made in section 4.

2.3 Long Run Impact of Real Options on Investment

We start off by examining a stylized investment model with one line of capital and flexible labour which is commonly used in the irreversible investment literature.

---

6This model of partial irreversibility is taken directly from Abel and Eberly (1996) and is a generalisation of the complete irreversibility case in Bertola (1988) and Dixit and Pindyck (1994) Chapter 11.
2.3.1 The Standard Model with a Single Line of Capital and Flexible Labour

The firm’s revenue function, \( R(K, P) \), in terms of its capital stock \( K \) and its demand conditions \( P \) is modelled as having the following form

\[
R(K, P) = \frac{1}{a} P^{1-a} K^a
\]

(2.1)

where this can be shown to nest a Cobb-Douglas production function and an iso-elastic demand curve. In this setup labour is assumed to be a completely flexible factor of production and has been optimized out of the revenue function. We assume that the firm’s demand conditions follow a Geometric Brownian motion process with drift \( \mu \) and variance \( \sigma^2 \). The firm is assumed to maximize the expected present value of sales revenues, minus the cost of buying capital at a price \( B \), plus the proceeds received from selling capital at a price \( S \), where \( B > S \). The optimal investment strategy then maximizes its total discounted profits

\[
\max_{\{I(s)\}} E_t \left\{ \int_t^\infty \exp^{-r(s-t)} \left( \frac{1}{a} P^{1-a}(s) K(s)^a ds - B dI^+(s) + S dI^-(s) \right) \right\}
\]

(2.2)

subject to

\[
dK(t) = -\delta K dt + I^+(t) - I^-(t)
\]

where \( r \) is the discount rate, \( I^+ \) denotes positive investment and \( I^- \) denotes negative investment, and \( \delta \) is the rate of capital depreciation.

Abel and Eberly (1996) demonstrate that the profit maximizing investment behavior can be described in terms of the firm’s current marginal revenue product of capital \( P^{1-a} K^{a-1} \), and an investment and disinvestment threshold, which is summarized in Table (2.1) below. These thresholds are

---

7For expositional simplicity we have left out the constant of proportion from the revenue function. This has no effect on any results.
represented by the investment and disinvestment user costs of capital, $C_I$ and $C_D$ respectively\(^8\), and two real options terms $\Phi_I > 1$ and $\Phi_D > 1$.

Table 2.1: The Threshold Behavior of Investment with Real Options.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest if:</td>
<td>$P^{1-a}K^{a-1} = C_I\Phi_I$</td>
</tr>
<tr>
<td>Do Nothing if:</td>
<td>$C_D/\Phi_D &lt; P^{1-a}K^{a-1} &lt; C_I\Phi_I$</td>
</tr>
<tr>
<td>Dis-Invest if:</td>
<td>$P^{1-a}K^{a-1} = C_D/\Phi_D$</td>
</tr>
</tbody>
</table>

After taking logs, these thresholds provide bounds for the logged capital stock under partial irreversibility, which using lower case to denote logged variables, can be stated as follows

$$p - \left( \frac{c_I + \phi_I}{1-a} \right) \leq k \leq p - \left( \frac{c_D - \phi_D}{1-a} \right)$$

(2.3)

To pinpoint the impact of real options we make a ceteris paribus comparison to the situation in which the firm acts as if it has no option to delay its investment. By modelling the firm as if it has a now or never investment choice we can turn off the real options effect but keep all other parameters and the evolution of demand constant\(^9\). Under this hypothetical alternative we know from Jorgenson (1963) that the firm will only invest when its marginal revenue product of capital is equal to its investment user cost of capital.

\(^8\)Jorgenson (1963) demonstrates that when capital costs a constant price $B$ to buy, the investment user cost of capital can be expressed as $C_I = (r + \delta)B$. This can be generalised for capital resale at a constant price $S$, to denote the disinvestment user cost of capital as $C_D = (r + \delta)S$.

\(^9\)This 'no real options' scenario can be equivalently described as the situation in which the firm always acts as if the current level of demand will extend into the future with complete certainty.
capital $C_t$, and only disinvest when its marginal revenue product of capital is equal to its disinvestment user cost of capital $C_D$. In the absence of real options the investment rule and capital stock, $K_{no}$, would satisfy the threshold optimality conditions expressed in table (2.2) below.

Table 2.2: The Threshold Behavior of Investment Without Real Options.

<table>
<thead>
<tr>
<th>Invest if:</th>
<th>$P^{1-a}K_{no}^{-1} = C_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do Nothing if:</td>
<td>$C_D &lt; P^{1-a}K_{no}^{-1} &lt; C_I$</td>
</tr>
<tr>
<td>Dis–Invest if:</td>
<td>$P^{1-a}K_{no}^{-1} = C_D$</td>
</tr>
</tbody>
</table>

After taking logs, these thresholds provide bounds for the logged capital stock with no real options under partial irreversibility, and can be stated as follows,

$$p - \frac{c_I}{1-a} \leq k_{no} \leq p - \frac{c_D}{1-a}$$ (2.4)

To evaluate the dynamics of investment we use the result that for continuous changes in the capital stock the instantaneous rate of investment is equal to the change in the log of the capital stock plus depreciation\(^{10}\)

$$\frac{I_t}{K_t} = d \log K_t + \delta$$ (2.5)

This allows us to use results on the long run growth rate of the capital stock to determine the long run rate of investment. Proposition 1 below demonstrates, perhaps surprisingly, that even though real options play an

\(^{10}\)It should be noted that since the evolution of these capital stocks is undertaking according to the threshold rules in Tables (2.1) and (2.2), they are 'variation finite' processes. Hence, they have zero quadratic variation and so do not require any Ito's Lemma adjustments to their drift rates when converted from logs to levels (see Harrison, 1990).
important role in the determination of the investment thresholds, they play no limiting role in long run investment.

**Proposition 1:** Real Options have no limiting effect on long run investment.

**Proof:** For partially irreversible investment combining the conditions (2.3) and (5.3) we find that the difference between the capital stock with real options and without real options is bounded by a finite constant,

\[
\frac{-\log \phi_I}{1 - a} \leq k - k_{no} \leq \frac{\log \phi_D}{1 - a}
\]

(2.6)

Hence, \(K\) and \(K_{no}\) have the same long run limiting\(^{11}\) growth rate since

\[
\lim_{T \to \infty} \frac{1}{T} |\log(K_{T+T}/K_t) - \log(K_{no,T+T}/K_{no,t})| = \lim_{T \to \infty} \frac{1}{T} |(log(K_{T+T}/K_{no,T+T}) - \log(K_t/K_{no,t})| \leq \lim_{T \to \infty} \frac{1}{T} \left| \frac{\log \phi_I}{1 - a} \right| \leq \lim_{T \to \infty} \frac{1}{T} \left| \frac{\log \phi_D}{1 - a} \right| = 0
\]

(2.7)\hspace{1cm} (2.8)\hspace{1cm} (2.9)

By (2.5) they also have the same long run rate of investment. For completely irreversible investment the distance between the capital stock with real options and without real options is, after any initial common investment episode, a fixed distance apart\(^{12}\),

\[
k - k_{no} = \frac{-\log \phi_I}{1 - a}
\]

(2.10)

\(^{11}\)This and all other limits in the paper are the standard deterministic limits.

\(^{12}\)For complete irreversibility the (positive investment) rules from Tables (2.1) and (2.2) are still optimal, so that during any common investment episode the levels of the capital stock with options and without options will be \(\frac{\log \phi_u}{1 - a}\) apart. Since subsequent depreciation imparts a linear trend to both capital stocks this \(\frac{\log \phi_u}{1 - a}\) gap between the two levels will persist from then onwards.
Hence, by the same logic as above they also have the same long run rate of investment.

These results are independent of any assumptions on the rate of capital depreciation, the rate of demand growth, the degree of uncertainty or the degree of irreversibility.

The intuition for this proof is that the capital stocks with and without real options are both contained within the real options investment thresholds. These thresholds are a fixed and bounded distance apart so that the gap between the two capital stocks is also fixed and bounded. Over time the importance of this fixed gap for long run investment tends to zero, as the evolution of demand becomes the first order determinant of investment. This is illustrated in figure (2.1), which plots in bold the investment profile for the capital stock with real options and without real options for a random 10 year realization of logged demand\textsuperscript{13}. Also plotted in figure (2.1) in feint are the investment and disinvestment thresholds for the capital stock with real options.

In figure (2.1) both the capital stocks with and without real options have been normalized to start off at one unit so that total cumulative investment can be measured from the current level of the capital stock. It can be seen that these two capital stocks evolve in a similar manner to each other. Figure (2.2) plots the evolution of these two capital stocks for the same demand process continued over a fifty year period from which the equivalence between

\textsuperscript{13}Logged demand is drawn from a Brownian process with 5\% drift and 20\% standard deviation. Returns to capital are assumed homogenous of degree 0.75, consistent with constant returns to scale production and a price elasticity of 4. Capital depreciates at 10\% per year, costs $1 per unit to buy and can be resold for $0.75 per unit. The firm’s annual discount rate is 10\%. 

28
Figure 2.1: The Short Run Level of the Logged Capital Stock with and without Real Options.

*Note:* The graph plots the evolution of the logged capital stock with real options (in bold) between its lower investment threshold (in feint) and its upper disinvestment threshold (in feint) in response to a randomly drawn demand process (see footnote 11 for details). Also plotted in bold is the evolution of the no real options logged capital stock in response to the said demand process.
Figure 2.2: The Long Run Level of the Logged Capital Stock with and without Real Options

Note: The graph plots the evolution of the logged capital stock with real options (the less variable line) in response to a randomly drawn demand process (see footnote 11 for details). Also plotted is the evolution of the no real options logged capital stock (the more variable line) in response to the same demand process.

the long run rates of growth is much clearer\textsuperscript{14}.

While the real options effect of uncertainty plays no role in long run investment, there is a specific case in which uncertainty does play a long run role by directly affecting the growth rate of logged demand. This effect is independent from real options.

\textsuperscript{14}For Ss models with \textit{fixed costs} of adjustment, for example as analyzed by Grossman and Laroque (1990), the long run rate of investment is also independent from any option value effects of uncertainty, since the investment rule takes on a similar threshold form.
Corollary to Proposition 1: Uncertainty can decrease (increase) expected long run investment, independently of real options and irreversibility, if logged demand is concave (convex) in the Brownian motion term.

Proof: From the definition of the thresholds for the level of capital stock with and without real options, equations (2.3) and equations (5.3), we can see that the long run growth rate of both levels of capital will be equal to the long run growth rate of logged demand,

\[
\lim_{T \to \infty} \frac{1}{T} \log(K_{T+t}/K_t) = \lim_{T \to \infty} \frac{1}{T} \log(K_{no,T+t}/K_{no,t}) = \lim_{T \to \infty} \frac{1}{T} \log(P_{T+t}/P_t)
\]

Jensen’s inequality states that long run growth of demand will be decreasing (increasing) in the variance of demand if \( \log P \) is concave (convex) in the underlying Brownian motion process. But, since this affects the level of the capital stock both with and without real options, this Jensen’s effect of uncertainty is independent of real options.

This explains Dixit and Pindyck’s (1994) result that the expected long run rate of investment is equal to \( \mu - \frac{\sigma^2}{2} \), and so reduced by higher uncertainty, since logged demand is concave in the level of demand. But this Jensen’s effect is not a robust theoretical prediction since it relies on the initial assumptions on the functional form of the demand process. For example, if we assume that logged demand, rather than the level of demand, is a Brownian motion process with mean \( \mu \) and standard deviation \( \sigma \), then this effect of uncertainty would disappear entirely.
2.3.2 A Model With Multiple Lines of Capital and Labour Adjustment Costs

The investment model outlined above, as is common in the literature, treats all types of capital within the firm as homogenous and labour as completely flexible because this makes the firm’s optimization problem analytically tractable. But even a cursory glance at establishment and firm level data will reveal that it is common for firms to operate at several different production locations, across industry classifications, and with different capital mixes and vintages. Furthermore, since at least the work of Oi (1962) it has been recognized that hiring and firing workers involves recruitment, training, reorganization and compensation costs, which makes labour a costly factor to adjust\textsuperscript{15}. We generalize the firm level production function to allow for $N$ separate lines of capital and $M$ types of labour and a more flexible demand process. This is done in a general way by considering the class of models which satisfy the following three assumptions\textsuperscript{16}:

1. The sales function is jointly concave and homogenous of degree $\lambda$ in all $N$ lines of capital and $M$ types of labour, where $\lambda < 1$. Individual lines of capital and types of labour within each plant are also complementary in production\textsuperscript{17}.

2. Lines of capital cost $\mathbf{B} = \{B_1, B_2, \ldots B_N\}$ to buy and can be resold for $\mathbf{S} = \{S_1, S_2, \ldots S_N\}$ where $0 < S < B$. labour can be hired at a

\textsuperscript{15}See, for example, Nickell (1986) for some estimates of the potentially substantial labour hiring and firing costs.

\textsuperscript{16}This class of models includes the Cobb-Douglas production and Brownian demand model we discussed previously in section (2.3.1).

\textsuperscript{17}This complementarity is defined such that for a production function $F(K_1, K_2, \ldots K_N; L_1, L_2, \ldots L_M)$ the marginal product of any individual factor of production is increasing in every other factor of production. For example, $\partial F(K_1, K_2, \ldots K_N)/\partial K_i$ would be increasing in all $K_j$, $j \neq i$. This condition is technically known as supermodularity and is described in more detail in Dixit (1997).
cost \( H = \{H_1, H_2, \ldots H_M\} \) and fired at a cost \( F = \{F_1, F_2, \ldots F_M\} \), where these costs include the present discounted value of all future wage payments, and \( 0 < F < H \).

3. The firm level demand shock has a multiplicative impact upon revenue and is generated by a stationary Markov process\(^\text{18}\).

Since firms may operate using several lines of capital and types of labour we have to generalize our threshold investment rule. Eberly and Van Mieghem (1997) demonstrate that the investment policy of a production plant with \( N \) lines of capital and \( M \) types of labour satisfying conditions (1) to (3) will be of a multi dimensional threshold form as characterized in Table (2.3).

Table 2.3: The \( N+M \) Dimensional Investment and Hiring Threshold Behavior.

<table>
<thead>
<tr>
<th>For Lines of Capital ( i = 1, 2, \ldots N )</th>
<th>For Types of labour ( j = 1, 2, \ldots M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest if: ( K_i = K_i^I )</td>
<td>Hire if: ( L_j = L_j^H )</td>
</tr>
<tr>
<td>Do Nothing if: ( K_i^D &lt; K_i &lt; K_i^P )</td>
<td>Do Nothing if: ( L_j^H &lt; L_j &lt; L_j^P )</td>
</tr>
<tr>
<td>Dis-Invest if: ( K_i = K_i^D )</td>
<td>Fire if: ( L_j = L_j^P )</td>
</tr>
</tbody>
</table>

For this more general set up we demonstrate in proposition 2 below that once again real options play no role in determining long run investment.

**Proposition 2:** For models satisfying assumptions (1) to (3) real options have no effect on the long run rate of investment.

*Proof:* In Appendix A

\(^{18}\)Stationarity implies that this process does not depend on time, while the Markov property implies that the future behaviour of the process depends on its present position but not on how it got there.
This suggests that even in quite general models of production and investment, after conditioning on demand growth to remove any Jensen’s inequality effects, uncertainty plays no role in determining the long run rate of investment.

2.4 The Short Run Impact of Real Options and Irreversibility on Investment and Labour Demand

The combination of real options and irreversibility do play an important role, however, in shaping the short run response to demand shocks. To explore this issue we first develop a methodology for characterizing these responses which is robust to any degree of aggregation. This is important because it enables us to make predictions on the dynamics of investment and hiring at the establishment, firm, industry, and macro level - thereby developing results that apply to data at the much lower levels of aggregation that is commonly used in empirical work. To ensure as wide a generality as possible we also maintain the multiple line of capital and inflexible labour model outlined by assumptions (1) to (3) in section (3.3). While our results will be stated for investment and hiring, to avoid repetition we will prove them only for investment, with the proof for hiring following by symmetry.¹⁹

Investment at the firm level will be the aggregate of investment in individual lines of capital within each plant. Hence firm-level investment will depend on the distribution of all lines of capital between their investment thresholds. To analyse this further we first consider the firm’s positive in-

¹⁹Note that we define the rate of labor hiring as the change in the logged labor force.
vestment expenditures. First, we define \( F(x) \) to be the cumulative density of capital within each firm\(^{20}\) that would respond to a positive demand shock of size \( x \geq 0 \). This implies, for example, that \( F(0) = 0 \) because all lines of capital will be either on or below their investment demand threshold and so will not respond to a size zero demand shock. In contrast, \( F(\Delta p) = 1 \), for example, would imply that all lines of capital (and hence all capital) would start investing after a (presumably large) demand shock of size \( \Delta p \). Figure (2.3) plots an example density function, \( f(x) = dF(x) \), of capital below its investment threshold with the shaded area representing \( F(d \log P) \), the lines of capital which would invest after a demand shock of size \( d \log P \).

Second, we can define, \( d \log \kappa(x, \Delta \log P) = \iota(x, \Delta \log P) \), the positive investment function for lines of capital at each point \( x \) of this cumulative density \( F(x) \) in response to a positive demand shock of size \( \Delta \log P \geq 0 \), as follows\(^{21}\)

\[
\begin{align*}
\iota(x, \Delta \log P) &> 0 \quad \text{if} \quad \Delta \log P > x \\
\iota(x, \Delta \log P) & = 0 \quad \text{if} \quad \Delta \log P \leq x
\end{align*}
\]

That is, \( \iota(x, \Delta \log P) \) is the change in the log of the capital stock for the lines of capital that would just start to invest in response to a shock of size \( x \), if they actually face a shock of size \( \Delta \log P \). The right hand side conditions follow from the definition of \( x \) as the smallest demand shock required to move capital at that position up to the investment threshold. This investment function will be increasing in the size of the demand shock so that \( \frac{\partial \iota(x, \Delta \log P)}{\partial \Delta \log P} \geq 0 \).

For firms with multiple lines of capital this investment function will also be

\(^{20}\)This can equivalently be defined at the proportion of all capital within each firm which would respond to a positive demand shock of size \( x \).

\(^{21}\)This investment function also depends on the whole distribution of capital \( F(.) \). So it could be written out fully for a position \( x \) and shock \( \Delta \log P \) as \( \iota(x, \Delta \log P, F(.)) \). However, since this reliance on the whole distribution \( F(.) \) does not affect the discussion of our main results, we use the abbreviated form \( \iota(x, \Delta \log P) \) to simplify our notation.
Figure 2.3: The Distribution of Capital from its Investment Threshold

Notes: The figure plots an example distribution of capital according to the size of the shock required to initiate investment. Thus, capital at point 0 will invest in response to a positive shock of any size, while the shaded area of capital up to point \( \log P \) would invest in response to a shock of size \( \log P \) (or greater).
convex (increasing at an increasing rate) due to the assumed supermodularity of capital in production, so that \( \frac{\partial^2 \pi(x, \Delta \log P)}{(\partial \Delta \log P)^2} \geq 0 \). Combining these two definitions, and using the approximation that \( d \log K = \frac{I}{K} \) where \( I \) is total firm investment and \( K \) is the total capital stock, we can characterize the firm’s investment rate given a demand shock of size \( \Delta \log P \) as \(^{22}\)

\[
\frac{I}{K} = \int_0^{\Delta \log P} \nu(x, \Delta \log P) dF(x) \quad (2.11)
\]

One important effect of real options and irreversibility is to reduce the investment and hiring response to a demand shock. This arises because firms will act more cautiously when capital and labour is partially irreversible and their market conditions are uncertain - any investment and hiring represents a gamble from which the firm can not easily extricate itself if conditions turn bad. This is noted in proposition 3 below.

**Proposition 3:** Real options and irreversibility will reduce the responsiveness of investment and hiring to demand shocks relative to complete reversibility.

**Proof:** In response to a positive demand shock of size \( \Delta \log P \) the investment response across all lines of capital will be

\[
\Delta \log(K) = \int_0^{\Delta \log P} \nu(x, \Delta \log P) dF(x) \quad (2.12)
\]

\[
\leq \int_0^{\Delta \log P} \frac{1}{1 - \lambda} (\Delta \log P - x) dF(x)
\]

\[
= \frac{1}{1 - \lambda} \int_0^{\Delta \log P} F(x) dx
\]

\[
\leq \frac{1}{1 - \lambda} \Delta \log P
\]

where the second line follows because \( \nu(x, \Delta \log P) \leq \frac{1}{1 - \lambda} (\Delta \log P - x) \) by the complementarity of capital in production (see appendix B for details), and

\(^{22}\)This can be justified by defining the firm level investment rate \( \frac{I}{K} \) to be the capital weighted average (using \( F(x) \)) of the investment rate of each individual line of capital.
the third line follows by integration by parts. If there is any capital lying below the investment threshold, so that $F(x) < 1$ for some $x > 0$, we obtain the strict inequality that $\Delta \log(K) < \frac{1}{1-\lambda} \Delta \log P$. To isolate the impact of real options and irreversibility we make the comparison to the hypothetical completely reversible level of capital, $K_R$. For this reversible capital we show in appendix B that

$$\Delta \log(K_R) = \frac{1}{1-\lambda} \Delta \log P$$

(2.13)

Combining (2.12) and (2.13) demonstrates that the investment response is lower under partial irreversibility than under complete reversibility.

---

The intuition for this result is that the region of inaction between the investment/disinvestment thresholds and the hiring/firing thresholds acts as a buffer against demand shocks. Within this region the response to shocks will be zero unless they are large enough to ensure that capital and labour are moved up against their investment and hiring thresholds. And even when the demand shock is large enough to ensure this happens the investment and hiring response will still be reduced by the zone of inaction.

Figure (2.4) plots an investment response, as an example, for a an economy comprised of a set of firms operating with a single line of capital and flexible labour, and where these firms are uniformly distributed between their investment and disinvestment thresholds. We can see that the investment response to a demand shock under partial irreversibility (the curved darker line) is always smaller than the investment response under complete reversibility (the straight lighter line). It can also be seen that while the demand response is always lower under partial irreversibility, this response is proportionally larger for larger shocks than for smaller shocks. This leads to a low but
Figure 2.4: The Investment Response to Demand Shocks When Capital is Uniformly Distributed below the Investment Threshold

Notes: The figure plots the investment response (in bold) of an economy of firms with partially irreversible capital which are uniformly distributed between their investment and disinvestment thresholds. Also plotted (in faint) is the investment response for this economy if capital were to be completely reversible.
increasing and convex investment and hiring response to demand shocks, as noted in proposition 4 below.

**Proposition 4:** Real options and irreversibility will lead to an increasing convex positive investment and hiring response to positive demand shocks.

*Proof:* Taking the first derivative of the investment response in (3.1) with respect to the demand shock yields a positive result, simply indicating that larger shocks lead to more investment

\[
\frac{\partial \Delta \log(K)}{\partial \Delta \log P} = \int_0^\Delta \log P \frac{\partial \nu(x, \Delta \log P)}{\partial \Delta \log P} dF(x) \geq 0
\]  

(2.14)

Taking the second derivative we see that the investment response is also increasing in the size of the shock

\[
\frac{\partial^2 \Delta \log(K)}{\partial (\Delta \log P)^2} = \int_0^\Delta \log P \frac{\partial^2 \nu(x, \Delta \log P)}{(\partial \Delta \log P)^2} dF(x) + \frac{\partial \nu(x, \Delta \log P)}{\partial \Delta \log P}|_{x=\Delta \log P} dF(\Delta \log P)
\]

\[\geq 0\]

where the first term is non-negative by the assumed complementarity of different lines of capital in production and the second term is non-negative by the non-decreasing nature of cumulative distribution functions.

This prediction matches the results of Caballero, Engel and Haltiwanger (1997) and Cooper and Haltiwanger (2000), who estimate employment and investment functions respectively on a panel of US establishment level data, and find a convex and increasing response.

Finally, in addition to the reduction in the short run response of investment and hiring to demand shocks, real options and irreversibility also lead
this response to be spread out over time, imparting rich and persistent dynamics to these processes. This is noted in proposition 5 below.

**Proposition 5:** Real Options and irreversibility will lead investment and hiring to be increasing in all past demand shocks.

*Proof:* In Appendix C

This can be interpreted in terms of the basic concept of pent up demand. Firms and industries with a history of strong recent demand growth will have a distribution of capital and labour lying close to their investment threshold and will display a strong investment and hiring response, while firms with a recent history of bad demand shocks will be less disposed to hire or invest. Empirically this will lead investment and labour demand to appear to respond to both current and lagged demand shocks. This matches the stylized facts from estimating firm and macro investment and labour demand equations. These display lagged responses to demand shocks usually spread over several quarters and years\(^3\).

### 2.5 Conclusion

In this paper we have shown that real options play no role in determining the long run rate of investment. This is demonstrated for both the standard model with Cobb-Douglas production and Brownian demand shocks, and also for a broader class of models with multiple lines of capital, inflexible labour and a generalized demand process. However, real options and irreversibilities are shown to play an important role in shaping the short run dynamics of investment and hiring. They reduce the short run response of investment

\(^{33}\text{See, for example, Chirinko (1993) and Hammermesh (1993).}\)
and hiring to current demand shocks and create lags in the response to past demand shocks.

The predictions of this model are consistent with the stylized facts from estimating firm, industry and macro level investment and labour demand equations. Hence, using real options to build a structural framework for estimating investment and labour demand should help to bridge the gap between what is often empirically preferable (reduced form models) and what is often theoretically preferable (their structural counterparts). And from the policy perspective, the time varying response elasticity of investment and employment over the business cycle can be explained by variations in the degree of macro uncertainty. Thus using measures of the current degree of macro uncertainty could improve the predictions of the response elasticities of investment and employment, helping policymakers to better model the effects of tax and interest rate changes.
2.6 Appendices

Appendix A

PROOF OF PROPOSITION 2:

For a plant with N+M lines of capital and types of labour satisfying assumptions (1) to (3) we define \( V(K, L, P) \) to be its value function given its current capital stock \( K = \{K_1, K_2, \ldots K_N\} \), labour force \( L = \{L_1, L_2, \ldots L_M\} \), and demand condition \( P \). By theorems 9.6, exercise 9.9 and theorem 9.10 respectively of Stokey et al. (1983) this value function will inherit the concavity and homogeneity properties of plant level sales and will be once continuously differentiable. We define \( U = \{B_1, \ldots B_N, H_1, \ldots H_M\} \) and \( D = \{S_1, \ldots S_N, F_1, \ldots F_M\} \) to be the combined N+M dimensional \{buy, hire\} and \{sell, fire\} prices of capital and labour. Following Eberly and Van Mieghem (1997) we can define the plant's investment and hiring thresholds by the vector of first differential of \( V(K, L, P) \) with respect to each line of capital and type of labour,

\[
D \leq \nabla V(K, L, P) \leq U
\]  

(2.15)

Since this value function is homogenous of degree one\(^{24}\) in \((K, L, P^{\frac{1}{1-\lambda}})\) its vector of first derivatives will be homogeneous of degree zero in \((K, L, P^{\frac{1}{1-\lambda}})\) so that this condition can be re-written as

\[
D \leq \nabla V(KP^{-\frac{1}{1-\lambda}}, LP^{-\frac{1}{1-\lambda}}, 1) \leq U
\]  

(2.16)

By the concavity of \( \nabla V(K, L, P) \) this defines a bounded continuation region such that

\[
P^{\frac{1}{1-\lambda}} \Gamma^I_K \leq K \leq P^{\frac{1}{1-\lambda}} \Gamma^P_K
\]  

(2.17)

\(^{24}\)By assumptions (1) to (3) sales is homogenous of degree \( \lambda \) in \( K \) and \( L \), and homogeneous of degree one in \( P \) (due to the multiplicative nature of the shock), and so sales is jointly homogenous of degree one in \((K, L, P^{\frac{1}{1-\lambda}})\).
\[ P^{1+\lambda} \Gamma_L^H \leq L \leq P^{1+\lambda} \Gamma_L^F \] (2.18)

where \( \Gamma_K^I, \Gamma_K^D \in R_N \) and \( \Gamma_L^H, \Gamma_L^F \in R_M \), with these being functions of the curvature of the value function and the cost of reversibility. Hence for capital we can write

\[ \frac{1}{1-\lambda} \log P + \log \Gamma_K^I \leq \log K \leq \frac{1}{1-\lambda} \log P + \log \Gamma_K^D \] (2.19)

By revealed preference these optimal investment and disinvestment thresholds contain the investment and disinvestment thresholds for a firm that ignores its option to delay the investment decisions so that

\[ \log \Gamma_K^I - \log \Gamma_K^D \leq \log K - \log K_{no} \leq \log \Gamma_K^I - \log \Gamma_K^D \] (2.20)

where \( \log K_{no} \) is the vector of no real options capital stocks.

Following the same logic as in proposition 1 we can show that \( \log K \) and \( \log K_{no} \) have the same long run limiting growth rate since

\[ \lim_{T \to \infty} \frac{1}{T} \left| \log(K_{T+t}/K_t) - \log(K_{no,T+t}/K_{no,t}) \right| = \lim_{T \to \infty} \frac{1}{T} \left| \log(K_{T+t}/K_{no,T+t}) - \log(K_t/K_{no,t}) \right| \]

\[ \leq \lim_{T \to \infty} \frac{1}{T} \log \Gamma^I + \log \Gamma^D \] (2.21)

\[ = 0 \]

\[ \nabla_K V(KP^{\frac{1}{1-\lambda}}, 1) = r \] (2.23)

Appendix B

NOTES FOR THE PROOF OF PROPOSITION 3:

From equation (2.15) above it can be seen that for completely reversible capital and labour we have
where \( r \) is the reversible cost of capital. Thus \( \log K = \frac{1}{1-\lambda} \log P + \log \Gamma_K \). The investment response to a demand shock can then be written as, \( \Delta \log K_i = \frac{1}{1-\lambda} \Delta \log P \), which, since this holds for the log of each line of capital, holds for the log of the total capital stock,

\[
\Delta \sum_{i=1}^{N} \log K_i = \frac{\sum_{i=1}^{N} dK_i}{\sum_{i=1}^{N} K_i} = \frac{1}{1-\lambda} \Delta \log P. \tag{2.25}
\]

To show that

\[
I(x, \Delta \log P) \leq \frac{1}{1-\lambda} (\Delta \log P - x). \tag{2.26}
\]

it is sufficient to note that under irreversibility every line of capital and type of labour may not be adjusting, and because these are supermodular (complementary) in production, the investment response to a demand shock at the investment threshold will be less than the reversibility case where all factors of production would adjust. This inequality will be strict if production is strictly supermodular and some factors do not adjust fully.

Appendix C

PROOF OF PROPOSITION 5:

The approach of this proof is to demonstrate that for any demand shock in period \( t \) a lagged positive demand shock in any period \( t - s, s > 0 \), will increase investment in period \( t \). And the larger the demand shock in period \( t-s \) the larger the increase in investment in period \( t \). Thus current investment will be an increasing function of past demand shocks.

Suppose in the absence of a some past demand shock the cumulative density of capital below the investment thresholds has the form \( F(x) \). The
investment function at each point $x$ in response to a demand shock $\Delta \log P_t$ takes the value $\ell(x, \Delta \log P_t, F(.))$, where we use the fuller notation which accounts for the impact of the distribution of capital on the investment response of each line of capital. Since all lines of capital are (weakly) complimentary in production the investment function will be weakly increasing in all weakly decreasing transformations of $F(.)$ across its support.

Now consider a counterfactual in which the firm experienced a shock in period $t - s$ of magnitude $\Delta \log P_{t-s} > 0$ so that the distribution below the investment threshold is now characterised by $\widetilde{F}(.)$. We can then define the new investment function at each point $x$ on the old cumulative density function by $\ell(x, \Delta \log P_t, \widetilde{F}(.))$. For each point $x$ on the old cumulative density function this new investment function must be greater or equal to than the old investment function since $\widetilde{F}(.) \leq F(.)$ across the support of $x$, because the lagged demand shock will have moved all lines of capital closer to their investment thresholds. Hence, using our characterization for investment we can write

$$\Delta \log K = \int_{0}^{\Delta \log P_t} \ell(x, \Delta \log P_t, \widetilde{F}(.))dF(x)$$

where $g(F(.), \widetilde{F}(.)) \geq 0$ is a positive function reflecting the impact of past demand shocks on the density of capital which will invest today. Hence, current investment is increasing in lagged demand shocks. Since $\ell(x, \Delta \log P_t, \widetilde{F}(.))$ is increasing in negative transformations of $\widetilde{F}(.)$, and $\widetilde{F}(.)$ is decreasing in the size of the lagged demand shock, current investment will display a larger re-
spose to larger lagged demand shocks. Since this result holds by symmetry for negative demand shocks the proof is complete.
Chapter 3

The Dynamics of Investment under Uncertainty

3.1 Abstract

We derive robust predictions on the effects of uncertainty on short run investment dynamics in a broad class of models with (partial) irreversibility. When their environment becomes more uncertain firms become more cautious and less responsive to demand shocks. This result contrasts with the long run analysis, in which the effect of real options on the level of the capital stock is ambiguous. An investment model is estimated to test these theoretical predictions using a panel of UK firms and a stock returns-based measure of uncertainty. As predicted we find that uncertainty reduces firms' responsiveness to demand shocks.
3.2 Introduction

The standard approach to modelling investment under uncertainty considers a firm operating a single production process and using a homogeneous capital good\(^1\). Investment decisions are assumed to be (partially) irreversible and market demand uncertain. This generates real options on the investment decision and a separation of the thresholds for investment and disinvestment, with no investment undertaken in between these thresholds. Even low levels of uncertainty and irreversibility can lead these thresholds to be significantly spaced apart in relation to their positions under complete certainty and costless reversibility, changing the optimal investment behaviour of firms from being smooth and continuous to one that is lumpy and frequently zero.

At first sight firm-level investment series appear too smooth to be consistent with these models of investment under uncertainty. But in micro establishment-level data, like the US Longitudinal Research Database (LRD) and the UK Annual Respondents’ Database (ARD), such lumpy investment with frequent zeros is observed, particularly for smaller plants\(^2\). This suggests that observations with zero investment at the firm level occur infrequently simply because of aggregation across multiple investment decisions. This is not surprising - firms are often observed to operate multiple production lines, plants and subsidiaries, each employing many types of capital goods. If these processes are not perfectly correlated due to idiosyncratic shocks and heterogeneous technologies then aggregation will smooth away much of the lumpiness from a firm-level series for total investment. Nevertheless uncertainty will still play an important role in determining firm-level investment.

\(^1\)See Bertola (1988), Pindyck (1988) and Dixit and Pindyck (1994), for example.
through its effects on the investment decisions for the individual types of capital. This has been shown in a number of papers on macro investment and consumption which demonstrate that aggregation does not diminish the effects of lumpy micro level behaviour\(^3\).

We develop a theoretical approach which identifies some robust predictions on firm-level investment dynamics which can be recovered after aggregation across multiple capital inputs, and we test these predictions empirically using firm-level panel data. Our main prediction is that uncertainty does play an important role in determining the *short run response* of investment to changes in market demand, whether or not uncertainty has any effect on the level of the capital stock in the longer term. Higher levels of uncertainty increase the real option values associated with investment and disinvestment and so make firms more cautious in responding to changes in their market environment. The presence of (partial) irreversibility and uncertainty also leads to non-linear investment dynamics with an increasing marginal investment response to larger demand shocks. This is potentially important because the dynamic response of firms to tax incentives and interest rates will depend on the uncertainty in their environment and the size of the stimulus. Since uncertainty and demand shocks have important cross sectional and time series variability, this also provides a possible explanation for the parameter instability within and across samples that has often been reported in the context of empirical investment equations.

Since the modelling strategy of this paper is to derive robust predictions on firm-level investment behaviour, we avoid making strong assumptions about the nature of production functions or the firm’s demand environment.

\(^3\) See, for example, Caballero (1993) and Eberly (1994) on aggregation across consumer durables, and Bertola and Caballero (1994), Caballero and Engel (1999), Cooper et al. (2000) and Attanasio et al. (2000) on aggregation across plant level investment.
Our underlying model encompasses a wide class of production functions and stochastic processes, and nests the standard Cobb-Douglas and Brownian motion assumptions found in the investment literature. This generality does not lead to a closed form analytical result but does provide a broad characterisation of investment behaviour. Using this framework we also demonstrate an important result for our empirical investigation - that a temporary increase in uncertainty increases the distance between the investment and disinvestment thresholds.

We test these predictions empirically using firm-level panel data and GMM estimates of dynamic investment equations. The firm level is attractive relative to the plant level in that we observe useful measures of uncertainty, and relative to the macro level in that we observe significant variation across firms in the level of uncertainty. Our theory suggests that higher uncertainty will reduce the response of firms to demand shocks. We add an additional interaction term between uncertainty and sales growth to our firm-level investment equations to test for this ‘caution effect’ of uncertainty and its role in generating time and firm varying investment parameters. We find this interaction term to be highly significant in our empirical analysis. In addition, we also include a quadratic term in sales growth to test for the predicted non-linear response of investment to demand shocks, and again find this to be highly significant. We also generate an artificial panel by simulating our theoretical model, and confirm that these theoretical predictions are detected by our estimation strategy applied to the simulated data.

The plan of the paper is as follows. Section 2 introduces the standard single line of capital model of investment under uncertainty and discusses the threshold behaviour this implies. In section 2.2 this is generalised to a multiple capital goods model with more general demand shocks. Section 2.3
introduces a new result on the impact of a temporary increase in the level of uncertainty, which temporarily increases the gap between the investment and disinvestment thresholds. Section 2.4 discusses aggregation over production plants to the level of total firm investment, and develops a second order approximation to the resulting investment behaviour. Our empirical strategy is outlined in section 3 and our investigation of the properties of this strategy using simulated data is discussed. Section 4 discusses the uncertainty measure and firm-level data, while section 5 reports our main empirical results and evaluates the quantitative impact of uncertainty on investment dynamics. Some concluding remarks are made in section 6. Technical and data appendices then follow.

3.3 Modelling Investment Under Uncertainty

The literature on investment under uncertainty predicts threshold investment behaviour\(^4\). Abel and Eberly (1996) examine a model of partial irreversibility and demonstrate that the solution to this can be fully characterized in terms of the firm's concentrated marginal revenue product of capital\(^5\), \(P^{1-a}K^{a-1}\) - where \(P\) is a demand term, \(K\) is capital and \(0 < a < 1\) - and its lower disinvestment and upper investment thresholds. These investment thresholds can be represented by the standard Jorgensonian user cost of capital for buying and selling capital, \(b\) and \(s\) respectively\(^6\), an investment real options

\(^4\)See, for example, Pindyck (1988) for fully irreversible continuous investment, Dixit (1989) for partially irreversible discrete investment, Bertola and Caballero (1994) for investment with stochastic demand and capital prices and Dixit and Pindyck (1994) for a general survey of the literature.

\(^5\)Labour and other inputs, which are assumed to be fully flexible in this model, have been optimised out of the concentrated marginal revenue product of capital.

\(^6\)Even under certainty the user cost for buying capital will be above the user cost for selling capital in a partial irreversibility framework where the sale price of capital is assumed to be below the purchase price.
term $\phi_I > 1$, and a disinvestment real options term $\phi_d > 1$. Investment only takes place when the marginal revenue product of capital hits the upper threshold and disinvestment only takes place when it hits the lower threshold. This investment policy is summarized in Table (3.1).

<table>
<thead>
<tr>
<th>Invest if:</th>
<th>$P^{1-a}K^{a-1} \geq b \times \phi_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do nothing if:</td>
<td>$s/\phi_D &lt; P^{1-a}K^{a-1} &lt; b \times \phi_I$</td>
</tr>
<tr>
<td>Disinvest if:</td>
<td>$P^{1-a}K^{a-1} \leq s/\phi_D$</td>
</tr>
</tbody>
</table>

Firms will undertake sporadic bursts of investment to ensure their capital stock stays between corresponding disinvestment and investment thresholds, with these thresholds being functions of model parameters, such as the degree of uncertainty, irreversibility, and also the current state of demand. Threshold investment and consumption models of this type have already been successfully estimated on a variety of data sets. For example, Caballero, Engel and Haltiwanger (1995), Cooper, Haltiwanger and Power (1999) and Attanasio and Pacelli (2000) estimate threshold investment models on US and UK plant level data, while Eberly (1994) and Attanasio (2000) have estimated threshold models of durable consumption using US car purchasing data. 7

3.3.1 A Multi Plant Model of Investment Under Uncertainty

Investment series for firms and large plants, however, appear to be too smooth and lacking in zero investment observations to be directly consistent with the

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7The non-convexities in these papers driving lumpy behaviour and zeros are generally fixed costs rather than partial irreversibilities, which also lead to threshold investment behaviour.
basic form of the threshold model. Table (3.2) reports the frequency of zero investment episodes for annual firm-level data (our sample of quoted UK firms), establishment-level data (a plant or multi-plant production facility from the UK Census of Production), single plant establishment data, and small single plant establishment data (less than 250 employees). There is a clear picture of more frequent zero investment episodes for individual types of capital goods and less aggregated production units. This suggests that aggregation across types of capital goods, production units and production sites obscures the zero investment observations at the firm level, with this aggregation also present in the establishment-level data.

Table 3.2: Zero Investment Episodes: Frequency (%).

<table>
<thead>
<tr>
<th></th>
<th>Buildings &amp; Land</th>
<th>Plant &amp; Machinery</th>
<th>Vehicles</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms</td>
<td>5.9</td>
<td>0.1</td>
<td>n.a.</td>
<td>0.1</td>
</tr>
<tr>
<td>Establishments (all plants)</td>
<td>46.8</td>
<td>3.2</td>
<td>21.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Single Plants</td>
<td>53.0</td>
<td>4.3</td>
<td>23.6</td>
<td>2.4</td>
</tr>
<tr>
<td>Small Single Plants</td>
<td>57.6</td>
<td>5.6</td>
<td>24.4</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 3.3: Simultaneous Investment and Disinvestment: Frequency (%).

<table>
<thead>
<tr>
<th></th>
<th>Firms</th>
<th>Establishments (all plants)</th>
<th>Single Plants</th>
<th>Small Single Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>48.0</td>
<td>28.2</td>
<td>23.3</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Firm-level data (11,098 obs.) from Extel UK and Datastream. Establishment-level data (46,689 obs.) from UK ARD Census of Production data (see Reduto dos Reis, 1999). Single plants (20,907 obs.) are single plant establishments. Small single plants (15,277 obs.) are those with less than 250 employees.

Given that firms invest in many kinds of capital goods and may have multiple plants, we need to develop an approach which can deal with this aggregation. This allows us to test for the predicted effects of uncertainty on investment using firm-level panel data, which has a number of advantages,
including the provision of a wealth of financial variables, which are not avail­able in establishment or industry-level data. These allow us to control for financial factors such as cash flow and debt. Firm-level data also provide a convenient proxy for uncertainty, in the form of the volatility of the firm’s daily share returns, which can be measured on a firm-year basis. The firm level is also often the focus of economic and policy interest in investment, which provides another motivation for using firm-level data. The empirical procedure we develop, however, could also be applied to other aggregated data sets at the industry or macro level.

To deal with this aggregation we generalise the firm-level production func­tion to allow for $N$ separate lines of capital. These lines of capital could be considered as single production projects in separate locations, operating independently and each employing a single capital input as, for example, in the standard model discussed above. Alternatively these lines of capital might be part of a broader production process employing multiple capital inputs, with operating decisions taken at the more aggregated process level. To deal with this potential ambiguity we define the ‘plant’ level to be “a production unit for which the solution of the optimisation problem can be identically undertaken at the level of that unit or at the level of the firm”\(^8\). For lines of capital which operate independently the plant level will be equal to the line of capital level, whilst for lines of capital in multi-line production processes

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\(^8\)An equivalent definition of plants could be made in terms of marginal separability. Plants are defined so that their marginal revenue products of every line of capital employed are separable from lines of capital in other plants. Firms may have between 1 and $N$ plants. This definition is not really restrictive, since all real-world plants can be considered as one model plant if this separability is not satisfied. Allowing for multiple plants allows us to rationalise firms that simultaneously invest and disinvest in our model.
the plant will be a larger more aggregated concept. This definition simply allows us to separate the firm’s overall production and investment decisions between its individual plants and then aggregate across these.

Production plants are assumed to be subject to productivity and demand shocks which are (at least partly) idiosyncratic, whilst firm-level demand shocks affect all plants within the firm. It will be seen that this results in two distinct types of aggregation taking place:

1. Aggregation across related lines of capital *within* each production plant,

   and

2. Aggregation across *separate* production plants within the firm.

Modelling firm-level investment behaviour then involves a two stage process. In the first step, undertaken in section (3.3.2), we characterise the nature of optimal investment at the plant level. In the second step, undertaken in section (3.3.4), we derive the implications for firm-level investment from our characterisation of plant-level investment.

### 3.3.2 Optimal Plant-Level Investment Behaviour

The general class of plant-level models to which our predictions apply is defined by assumptions (1) to (3) below:

1. The sales revenue function is continuously differentiable, jointly concave and homogeneous of degree $\lambda$ in all lines of capital, where $\lambda < 1$.

   Whilst $\lambda$ is fixed for plants within the same firm it may vary between
firms. Individual lines of capital within each plant are supermodular in production\textsuperscript{9}.

2. Adjustment costs are weakly convex and kinked at zero investment due to partial irreversibilities.

3. The firm-level demand shock and the plant-level shocks have a multiplicative impact on sales revenue and are generated by a stationary first order Markov process\textsuperscript{10}.

These conditions encompass a general class of production functions and stochastic processes, for example nesting the standard Cobb-Douglas and Brownian motion approach found in the investment and uncertainty literature and outlined above. Since plants may operate using several types of capital we have to generalise our threshold investment rule from one type of capital to multiple capital inputs. Eberly and Van Mieghem (1997) show that the investment policy of a production plant with \( N \) lines of capital satisfying conditions (1) to (3) will be of a multi-dimensional threshold form as characterised in Table (3.4) below.

In the absence of depreciation, the most flexible line of capital with the lowest adjustment cost would adjust first to a demand shock, so that its adjustment cost will determine the width of the no-investment threshold.

\textsuperscript{9}Supermodularity is defined such that for a plant-level production function \( F(K_1, K_2, \ldots, K_N) \) the marginal product of any individual line of capital is increasing in the other lines of capital - that is \( \partial F(K_1, K_2, \ldots, K_N)/\partial K_i \) is increasing in \( K_j \), \( j \neq i \). See also Dixit (1997).

\textsuperscript{10}Stationarity implies that this process does not depend on time whilst the first order Markov property implies that only current information is needed for forecasting future demand.
Table 3.4: N Dimensional Threshold Behaviour.

<table>
<thead>
<tr>
<th>For Lines of Capital $i = 1, 2, \ldots N$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest if: $K_i \leq K_i^I$</td>
<td></td>
</tr>
<tr>
<td>Do nothing if: $K_i^I &lt; K_i &lt; K_i^D$</td>
<td></td>
</tr>
<tr>
<td>Disinvest if: $K_i \geq K_i^D$</td>
<td></td>
</tr>
</tbody>
</table>

for the plant as a whole. Thus infrequently observed investment zeros at the plant level could be consistent with some lines of capital being fully irreversible as long as other lines of capital in the plant are relatively costless to adjust. In the presence of differential depreciation rates this probability of observing zero investment may be even lower since some lines of capital may be near their investment boundaries due to natural depreciation drift.

Hence plant-level investment is smoothed by the aggregation over different lines of capital even though investment in each line of capital may be lumpy and frequently zero. This may explain the relatively smooth nature of investment series with few zeros in establishment-level data such as the US LRD and UK ARD. This aggregation provides one route for firm-level investment dynamics to be smoothed relative to the familiar single project model, even for single plant firms. Before considering aggregation over multiple plants to characterise total firm investment in section (3.3.4), we first consider the effects of a temporary change in uncertainty on these plant-level investment thresholds.
3.3.3 Effects of Uncertainty and Changes in Uncertainty

In the case of a single capital input, Abel and Eberly (1996) show that a higher level of uncertainty will be associated with a wider gap between the investment and disinvestment thresholds. This comparative static result extends straightforwardly to the context of a plant with multiple capital inputs discussed above. However to identify the effects of uncertainty on investment empirically we will typically rely on measures of uncertainty that vary over time for the same firm. Policy interest also centres on the effects of changes in the level of uncertainty. It would therefore be useful to obtain a theoretical characterisation of the effect on a firm’s investment policy of a change in the level of uncertainty that it faces. We develop a new result, Proposition 1 below, which takes a step in this direction by analysing the case of a temporary change in the level of uncertainty\textsuperscript{11}. In particular, we consider the effects of a change in the distribution of demand shocks in the current period, holding constant the distribution of demand shocks in all future periods.

For the multiple lines of capital model outlined above in section (3.3.2), Proposition 1 demonstrates that such a temporary increase in uncertainty leads to a similarly temporary increase in the gap between the investment and disinvestment thresholds. To do this we use a broader concept of uncertainty, second order stochastic dominance, which can be defined by: "If

\textsuperscript{11}Hassler (1996) has previously examined the effects of time varying uncertainty on thresholds in a fixed cost Ss model, demonstrating that higher uncertainty increases the between threshold distance. While these results are instructive they can not be extended to our general framework.
the distribution \( F_A(P) \) stochastically dominates \( F_B(P) \) then for any outcome \( P_A \) from the dominating distribution the outcome from the dominated distribution is equal to that plus a mean preserving spread, so that \( P_B = P_A + \varepsilon \), where the random variable \( \varepsilon \) has mean zero and positive variance. For the standard class of Brownian motion models the definitions of second order dominance and variance are equivalent, while for more general distributions second order dominance always implies lower variance whilst the converse is often true although there can be exceptions\(^1\).

**Proposition 1:** An increase in demand uncertainty, defined in terms of second order stochastic dominance, in the current period, holding constant demand uncertainty in all future periods, will weakly increase the gap between the disinvestment and investment thresholds for all lines of capital in the current period.

Proof: See Appendix A.

This is the caution effect whereby increases in uncertainty increase the probability of making expensive mistakes and lead the plant to pursue a more cautious investment policy.

### 3.3.4 Firm-Level Investment Behaviour

Investment at the firm level will be the aggregate of investment in individual lines of capital within each plant. Hence firm-level investment will depend on the distribution of all lines of capital between their investment thresholds. To analyse this further we first consider the firm’s positive investment expen-

\(^1\)For further results and details of stochastic dominance, see Rothschild and Stiglitz (1970).
ditutes. First, we define \( F(x) \) to be the cumulative density of capital within each firm\(^{13}\) that would respond to a positive demand shock of size \( x \geq 0 \).

This implies, for example, that \( F(0) = 0 \) because all lines of capital will be either on or below their investment demand threshold and so will not respond to a size zero demand shock. In contrast, \( F(\Delta p) = 1 \), for example, would imply that all lines of capital (and hence all capital) would start investing after a (presumably large) demand shock of size \( \Delta p \).

Second, we can define, \( d \log \kappa(x, \Delta p) = \iota(x, \Delta p) \), the positive investment function for lines of capital at each point \( x \) of this cumulative density \( F(x) \) in response to a positive demand shock of size \( \Delta p \geq 0 \), as follows\(^{14}\)

\[
\begin{align*}
\iota(x, \Delta p) &= \begin{cases} 
0 & \text{if } \Delta p < X \\
1 & \text{if } \Delta p \geq X
\end{cases}
\end{align*}
\]

That is, \( \iota(x, \Delta p) \) is the change in the log of the capital stock for the lines of capital that would just start to invest in response to a shock of size \( x \), if they actually face a shock of size \( \Delta p \). The right hand side conditions follow from the definition of \( x \) as the smallest demand shock required to move capital at that position up to the investment threshold. This investment function will be increasing in the size of the demand shock so that \( \frac{\partial \iota(x, \Delta p)}{\partial \Delta p} \geq 0 \). For firms with multiple lines of capital this investment function will also be convex (increasing at an increasing rate) due to the assumed supermodularity of capital in production, so that \( \frac{\partial^2 \iota(x, \Delta p)}{(\partial \Delta p)^2} \geq 0 \). Combining these two definitions, and

\(^{13}\)This can equivalently be defined at the proportion of all capital within each firm which would respond to a positive demand shock of size \( x \).

\(^{14}\)This investment function also depends on the whole distribution of capital \( F(.) \). So it could be written out fully for a position \( x \) and shock \( \Delta p \) as \( \iota(x, \Delta p, F(.) \). However, since this reliance on the whole distribution \( F(.) \) does not affect the discussion of our main results, we use the abbreviated form \( \iota(x, \Delta p) \) to simplify our notation.
using the approximation that \( d \log K = \frac{I}{K} \) where \( I \) is total firm investment and \( K \) is the total capital stock, we can characterize the firm's investment rate given a demand shock of size \( \Delta p \) as\(^{15}\)

\[
\frac{I}{K} = \int_0^{\Delta p} \mu(x, \Delta p)dF(x) 
\] \hspace{1cm} (3.1)

Without imposing additional restrictions on the production function and the stochastic demand and productivity processes this investment function has no closed form analytic solution. But for the general class of investment problems defined by conditions (1) to (3) in section (3.3.2) we can derive predictions about firm-level investment dynamics from a second order Taylor expansion of investment around uncertainty and the demand shock.

**Short Run Dynamics**

We first consider the short run response of firm-level investment to demand shocks and changes in the level of uncertainty. In particular we can characterise the sign of these responses. To derive these empirical implications we take a Taylor expansion of investment defined in (3.1) in terms of firm-level demand shocks and uncertainty.

The first derivative of positive investment with respect to a demand shock is positive reflecting the impact on lines of capital already at the investment margin

\[
\frac{\partial I}{\partial \Delta p} = \int_0^{\Delta p} \frac{\partial \mu(x, \Delta p)}{\partial \Delta p}dF(x) \geq 0 
\] \hspace{1cm} (3.2)

\(^{15}\)This can be justified by defining the firm level investment rate \( \frac{I}{K} \) to be the capital weighted average (using \( F(x) \)) of the investment rate of each individual line of capital.
The second derivative of positive investment with respect to a demand shock is also positive, 

$$\frac{\partial^2 I}{K \partial (\Delta p)^2} = \int_0^{\Delta p} \frac{\partial^2 \nu(x, \Delta p)}{(\Delta p)^2} dF(x) + \frac{\partial \nu(x, \Delta p)}{\partial \Delta p} \bigg|_{\Delta p} dF(\Delta p) \geq 0$$

because the first term is non-negative by the assumed supermodularity of different lines of capital in production and the second term is non-negative by the non-decreasing nature of cumulative distribution functions. Thus the firm's investment rate is an increasing and convex function of the demand shock.

The first derivative of positive investment with respect to a temporary increase in uncertainty will be negative. This is because, as Proposition 1 notes, higher uncertainty increases the real option value associated with investment and raises the investment threshold. So with higher uncertainty the new investment function, \(\nu'(x, \Delta p)\), defined according to the old distribution of capital \(F(x)\), will be less than the old investment function, so that \(\nu'(x, \Delta p) \leq \nu(x, \Delta p)\). Hence we can write,

$$\frac{\partial I}{K \partial \sigma} = \lim_{\Delta \sigma \to 0} \int_0^{\Delta p} \frac{\nu'(x, \Delta p) - \nu(x, \Delta p)}{\Delta \sigma} dF(x) \leq 0 \quad (3.4)$$

where the second line follows because limits preserve weak inequalities.\(^{16}\)

\(^{16}\)Proposition 1 refers to the increase in the gap between the investment and disinvestment thresholds in terms of a stochastic dominance relationship between two distributions, rather than in terms of a continuous function of uncertainty. So this derivative should be interpreted as a Radon-Nikodym functional derivative on the probability space of demand distributions, rather than the usual calculus concept. However, to avoid unnecessary notation and complexity, and because this does not affect the empirical interpretation of this derivative, we will not elaborate any further on this point but refer the interested reader instead to Bartle (1966).
Finally, considering the cross product term in the Taylor expansion we see that the cross effect of a positive demand shock and a temporary increase in uncertainty on positive investment is negative. This is because higher levels of uncertainty raise the investment threshold leading to less lines of capital investing. This reduces the investment response for all lines of capital that are investing due to the supermodularity of capital in production, so that \( \partial v'(x, \Delta p) / \partial \Delta p \leq \partial v(x, \Delta p) / \partial \Delta p \). Combining this relationship and (3.4) we can state that

\[
\frac{\partial^2 I}{K} / \partial \sigma \partial \Delta p = \lim_\sigma \to 0 \int_0^{\Delta p} \frac{\partial v'(x, \Delta p) / \partial \Delta p - \partial v(x, \Delta p) / \partial \Delta p}{d\sigma} dF(\Delta p) \leq 0
\]

where the second line again follows by the preservation of weak inequalities in the limit.

Thus far we have only considered the effects on positive investment expenditures. Deriving the corresponding results from the equivalent expansion for disinvestment uses a similar approach and a summary of both sets of results is presented in Table (3.5) below.

Table 3.5: The Short Run Sign of Demand Shocks (\( \Delta p \)) and Uncertainty (\( \sigma \)) on Investment.

<table>
<thead>
<tr>
<th>Taylor Expansion Terms</th>
<th>( \Delta p )</th>
<th>( \Delta p^2 )</th>
<th>( \sigma \Delta p )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Investment</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Disinvestment</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
As we saw in Table (3.3), almost all observations on total firm-level investment in our data cover firms that are simultaneously investing and disinvesting. The total investment series we model measures gross investment expenditures net of sales of capital goods. Thus only the direct demand shock term and the uncertainty-demand shock interaction term have an unambiguous prediction on measured investment dynamics. A positive demand shock should result in more investment (or less disinvestment), and a higher level of uncertainty should reduce the response of investment (or disinvestment) to a given demand shock. For this reason we focus on testing these predictions in our empirical investigation. In principle, the short run signs of accelerating demand and the level of uncertainty on measured investment dynamics are ambiguous. However, because of the effects of positive sales growth and capital depreciation, the vast majority of the total investment observations are positive in our firm-level data set (about 96% of observations). We would thus expect the short run effects of accelerating demand and higher uncertainty to be positive and negative respectively, consistent with the predictions for firms engaging in positive investment.

**Long Run Effects**

To investigate these predictions about the short run investment dynamics empirically, we need to control for longer run influences on the firm’s capital stock. To do this we exploit the results of Bloom (2000a) which demonstrate that the long run growth rates of the firm’s actual capital stock, $K_t$, and its hypothetical level under costless reversibility, $K_t^*$, will be equal, so that their levels will be cointegrated. Thus, the long run behaviour of the actual capital
stock under partial irreversibility can be modelled using its much simpler hypothetical value under complete reversibility plus a stationary deviation

$$\log K_t = \log K_t^* + e_t$$  \hspace{1cm} (3.6)

where $e_t$ is a stationary, autocorrelated error term bounded by the distance between the disinvestment and investment thresholds.

Notice that this does not imply that the actual capital stock and its hypothetical reversible level will be equal on average - there is no requirement for the deviation $e_t$ to be mean zero. Similarly this result does not rule out any particular long run relationship between uncertainty and the level of the capital stock. As has been stressed by Abel and Eberly (1999) and Caballero (1999), among others, the impact of higher uncertainty on the average capital stock level in the long run is theoretically ambiguous in models with partial irreversibility, since higher uncertainty retards downward as well as upward adjustments. For this reason we allow for the possibility of long run effects of uncertainty on the level of the capital stock, as well as the predicted effects on short run investment dynamics, in our empirical specification.

### 3.4 Empirical Specification

Since the distribution of projects between their disinvestment and investment thresholds is dependent on the past history of demand shocks, the empirical dynamics of investment will be rich and persistent. A convenient starting point for our empirical specification is an error correction model, which separates parameters describing the short run investment dynamics from those
describing the long run evolution of the capital stock\(^{17}\). Linear error correction models have been used in the aggregate investment literature by, for example, Bean (1981), and in the micro investment literature by Bond et al (1999).

The basic error correction model has the form

\[
\frac{I_t}{K_{t-1}} \approx \Delta \log K_t = \alpha_0 + \alpha_1(L) \Delta \log K_{t-1} + \alpha_2(L) \Delta \log K^*_t + \alpha_3 \left( \log K^*_{t-s} - \log K_{t-s} \right) + v_t \tag{3.7}
\]

where \(\alpha_1(L)\) and \(\alpha_2(L)\) are polynomials in the lag operator \((L)\). This is consistent, for example, with an ARMA approximation to the stationary error term in (3.6). Since the consistency of the Generalised Method of Moments (GMM) estimator we use depends on orthogonality between the residual error term \((v_t)\) and a set of suitably lagged instruments, our empirical dynamic specification is selected with this criterion in mind.

Our specification for firm \(i\)'s hypothetical capital stock under complete reversibility has the simple log-linear form

\[
\log K^*_t = \log Y_t + \gamma_1 \sigma_t + \gamma_2 \left( \frac{C_t}{K^*_t} \right) + A_i + B_t \tag{3.9}
\]

where \(Y_t\) is firm \(i\)'s sales in period \(t\), \(\sigma_t\) is a measure of uncertainty, \(\left( \frac{C_t}{K^*_t} \right)\) is cash flow scaled by the previous period's capital stock, \(A_i\) is an (unobserved) firm-specific effect and \(B_t\) is a time-specific effect, common to all firms.

\(^{17}\)It should be noted that a Q investment model, by assuming perfect competition and constant returns, assumes away any role for real options from the outset. Hence, while Abel and Eberly's (1994) generalisation of the Q model can be used to test for the presence of non-convex adjustment costs, because it assumes perfect competition and constant returns to scale, it rules out any real options effects by construction. In a similar manner the standard investment Euler equation, which does not permit the type of kinked adjustment costs that arises from partial irreversibility, also assumes away any role for real options from the outset.
For the baseline case in which $\gamma_1 = \gamma_2 = 0$, Proposition 2 below shows that this expression for the desired capital stock is consistent with our general model with multiple capital goods.

**Proposition 2**: The log of the capital stock under *complete reversibility* is equal to the log of sales plus a bounded term $Z_{it}$, so that we can write

$$
\log K^*_it = \log Y_{it} + Z_{it}
$$

(3.10)

*Proof*: See Appendix A

Allowing for time effects in (3.9) controls for variation in costs of capital to the extent that these are common to all firms. Firm effects allow for variation across firms in the elasticity of demand, and for some firm variation in relative prices. The uncertainty term allows for a possible long-run effect of uncertainty on the firm’s capital-sales ratio. The cash flow term allows for possible effects from liquidity constraints (cf. Fazzari, Hubbard and Petersen (1988)) or managers over-investing free cash flow (cf. Jensen (1986)). Whilst this is not our main interest in this analysis, many previous studies have found cash flow terms to be significant in reduced form investment equations. We therefore follow Guiso and Parigi (1999) in including such terms so as not to mistake uncertainty effects for omitted cash flow effects. We also found that the inclusion of cash flow terms was important for the validity of the orthogonality conditions we use in estimation.

Combining equations (3.7) and (3.9) gives a linear error correction model relating the current investment rate to lagged investment rates, current and lagged changes in sales, uncertainty and cash flow, and a lagged error correction term. We include additional interaction terms between uncertainty
and sales growth, and powers of sales growth, in order to test for the heterogeneous and non-linear investment dynamics predicted by our theoretical model. Starting from a more general specification, exclusion of insignificant terms led to our basic model

\[
\frac{I_{it}}{K_{i,t-1}} = \beta_1 \Delta y_{it} + \beta_2 (\Delta y_{it})^2 + \beta_3 (\sigma_{ut} \Delta y_{it}) + \beta_4 \Delta \sigma_{it} + \beta_5 \Delta \frac{C_{it}}{K_{i,t-1}} + \beta_6 (y - k)_{i,t-1} + \beta_7 \sigma_{i,t-1} + \beta_8 \frac{C_{it-1}}{K_{i,t-2}} + b_i + a_i + u_{it}
\] (3.11)

where \( k_{it} = \log K_{it} \) and \( y_{it} = \log Y_{it} \). The first four terms correspond to the influences on short run investment dynamics analysed in section (2.4.1). Our theoretical analysis predicts that \( \beta_1 > 0 \) and \( \beta_3 < 0 \), whilst it is likely that \( \beta_2 > 0 \) and \( \beta_4 < 0 \) given that our sample is dominated by firms with positive investment. We require \( \beta_6 > 0 \) for the estimated model to be consistent with ‘error correcting’ behaviour (i.e. a capital stock below its desired level to be followed eventually by upward adjustment), whilst the long run effect of uncertainty on the level of the capital-sales ratio (\( \beta_7 \)) is theoretically ambiguous.

Whilst the inclusion of squared sales growth and the uncertainty interaction term in this model tests the null hypothesis of a common, linear error correction specification against the non-linear and heterogeneous investment dynamics predicted by our theoretical analysis, it is nevertheless likely that the actual investment dynamics under the alternative would be more heterogeneous across both firms and time than this common parameters specification permits. Unfortunately, it is not possible to allow for arbitrary heterogeneity in slope coefficients in models with predetermined and endogenous covariates. At one level we can consider the included dynamics in (3.11) to
be removing a common component of the deviation term \(e_{it}\) from the long run specification (3.6). Provided the residual component \(u_{it}\) is orthogonal to our instruments, this would in principle allow us to identify common long run parameters of the specification for \(\log K^n_t\). However it should be noted that the status of the estimated short run dynamics under the alternative is less clear.

To confirm that our empirical test does have power to detect the short run dynamics implied by our theoretical model, Appendix C reports the results of applying this test using simulated data generated by a partial irreversibility model of the type analysed in section 2. These simulations indicate that the short run dynamics predicted in section (3.3.4) would be detected by this empirical analysis if the data were indeed generated by a model of this type. In particular, estimating a non-linear error correction model on the generated data, we find that the coefficient on the squared sales growth term \((\Delta y_{it})^2\) is significantly positive, and the coefficient on an uncertainty interaction term \((\sigma_i \Delta y_{it})\) is significantly negative. Moreover, the inclusion of a linear error correction term \((y - k)_{i,t-1}\) appears to be sufficient to control for the serial correlation found in static specifications, so that the estimated equation does not fail the serial correlation test or Sargan-Hansen test of overidentifying restrictions in samples of this size.

Estimation uses the system GMM estimator for dynamic panel data (see Blundell and Bond, 1998) which extends the standard moment conditions in the first-differenced GMM estimator subject to additional (testable) initial condition restrictions. The system estimator combines equations in first-differences, from which the firm-specific effects are eliminated by the trans-
formation; and equations in levels, for which the instruments used must be orthogonal to the unobserved firm-specific effects. The overidentifying restrictions are tested using a Sargan-Hansen statistic, and residual serial correlation tests are reported. A goodness of fit measure for our models is also provided, which is the squared correlation between the predicted level of the investment rate and the actual investment rate. This squared correlation between the actual and predicted variables is equivalent to the standard $R^2$ for OLS regressions and has been suggested as a goodness of fit measure for IV regressions (see for example Windmeijer (1995)).

### 3.5 Data

#### 3.5.1 The Measurement of Uncertainty

Although the model developed above discusses the response of investment to demand and productivity shocks, the notion of uncertainty we have in mind is much broader in scope. In reality firms will be uncertain about future prices, wages rates, exchange rates, technologies, consumer tastes and government policies. In an attempt to capture all factors in one scalar proxy for firm-level uncertainty, we follow Leahy and Whited (1998) in using the standard deviation of the firm’s daily stock returns, adjusted for gearing, denoted $\sigma_u$. This measure includes on a daily returns basis the capital gain on the stock, dividend payments, rights issues, and stock dilutions. Such a returns measure provides a forward looking proxy for the volatility of the firm’s environment which is implicitly weighted in accordance with the impact of these variables.
A stock returns-based measure of uncertainty is also advantageous because the data is accurately reported at a sufficiently high frequency to use an annual measure. When using homoskedastic diffusion processes the variance of the sample variance is inversely related to the sampling frequency. Our sampling frequency of about 265 recordings a year therefore suggests low sample variance. A disadvantage of using this measure is that the variability in stock market returns may not truly reflect fundamentals. Guiso and Parigi (1999), for example, use the variance implied by managers’ expected distributions of future variables, which they construct from a special survey. The results which we find below are qualitatively similar to theirs, even though we use different measures of uncertainty and, unlike them, we have panel data rather than a single cross section.

In the time series dimension (see Figure (3.1)) there is a spike in uncertainty in 1975 around the time of the first OPEC oil shock. There is as another peak in 1988 when annual measures are affected by the October 1987 stock market crash. In common with Davis and Haltiwanger (1992) we also find that these macro sources of uncertainty are less important than firm-specific idiosyncratic shocks. For example, in our data only 17% of the variance of our uncertainty measure is accounted for by macro shocks. Of

---

18To allow for possible effects of market-wide bubbles and fads we also calculated a second measure of uncertainty, using the standard deviation of the firm’s daily share returns normalized by the return on the FTSE All-Share index. Results using this normalized measure were very similar to those reported below, and are available on request from the authors. We also obtained qualitatively similar results whether we used the standard deviation or the variance as our measure of uncertainty.

19For example, Andersen and Bollerslev (1998) use high frequency exchange rate data with 288 recordings per period and calculate that the implied measurement errors are less than 2.5% of the true volatility.
the residual about half is permanent idiosyncratic differences between firms and half is transitory idiosyncratic shocks.

3.5.2 Investment and Other Accounting Data

The company data is taken from Datastream and consists of 672 manufacturing companies quoted on the UK stock market from 1972 to 1991. Investment in fixed capital assets is measured net of revenue from asset sales, which may nevertheless under record the value of disinvestment. Our capital stock mea-
sure is derived from the book value of the firm’s stock of net fixed assets, using the investment data in a standard perpetual inventory formula. Real sales are deflated using the aggregate GDP deflator. Cash flow is reported post-tax earnings plus depreciation deductions. Appendix B provides further details and Table (4.2) reports some summary statistics for the sample.

Table 3.6: Descriptive Statistics of 672 firms, 6019 observations.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>median</th>
<th>standard deviation</th>
<th>min.</th>
<th>max.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>total</td>
<td>within</td>
<td>betwn</td>
<td>total</td>
<td>within</td>
</tr>
<tr>
<td>$(I_t/K_{t-1})$ (investment rate)</td>
<td>0.128</td>
<td>0.093</td>
<td>0.13</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>$\Delta y_t$ (log real sales growth)</td>
<td>0.031</td>
<td>0.026</td>
<td>0.16</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>$\sigma_t$ (s.d. of share returns)</td>
<td>1.56</td>
<td>1.41</td>
<td>0.68</td>
<td>0.53</td>
<td>0.47</td>
</tr>
<tr>
<td>$(C_t/K_{t-1})$ (cash flow term)</td>
<td>0.18</td>
<td>0.140</td>
<td>0.16</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td>Employment</td>
<td>8,400</td>
<td>1,481</td>
<td>24,492</td>
<td>8,461</td>
<td>19,450</td>
</tr>
<tr>
<td>Observations per firm</td>
<td>11.3</td>
<td>11</td>
<td>4.7</td>
<td>0</td>
<td>4.7</td>
</tr>
</tbody>
</table>

As a preliminary step we present some basic descriptive regressions in Table (3.7). There is a strong negative correlation between investment rates and our measure of uncertainty which is illustrated in column (1). Column (2) then includes the growth of sales and its interaction with uncertainty. It is the latter variable which is key for our theoretical results. There is a standard accelerator effect of sales but, more importantly, the coefficient on sales growth is significantly lower for firms with more volatile share returns. In the final column we condition on cash flow as an additional regressor, which is highly significant. The interaction of sales growth with uncertainty remains negative and significant. The linear effect of uncertainty now reverses sign and is insignificant at conventional levels. Although these results do not control for endogeneity and firm-specific effects, it is interesting that our key
theoretical prediction is consistent with the raw data.

### Table 3.7: Descriptive Investment Regressions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (I_t/K_{t-1})^{20} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty (( \sigma_t ))</td>
<td>-0.015</td>
<td>-0.005</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Sales Growth (( \Delta y_t ))</td>
<td>0.453</td>
<td>0.338</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.023)</td>
<td></td>
</tr>
<tr>
<td>Uncert.*Sales Growth (( \sigma_t * \Delta y_t ))</td>
<td>-0.033</td>
<td>-0.035</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td>Cash Flow (( C_t/K_{t-1} ))</td>
<td></td>
<td></td>
<td>0.381</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.011)</td>
</tr>
</tbody>
</table>

### 3.6 Econometric Results

Table (3.8) contains our main econometric results estimated by GMM\(^{21}\). The first column contains a basic linear error correction model augmented to include cash flow variables. The error correction term is correctly signed and of a similar magnitude to others in the literature (e.g. Bond, Harhoff and Van Reenen, 1999). Current sales growth and both the level and the change in the cash flow variable are also significant determinants of investment. The second column adds in a non-linear term in squared sales growth which is positive and highly significant, consistent with our theoretical prediction, and noticeably improves the fit of the model. The third column includes our uncertainty variables - the change in uncertainty, the lagged level of uncertainty, and the interaction between uncertainty and sales growth. The level of uncertainty is negative but insignificant, which is consistent with the ambiguous effect of uncertainty on the level of the capital stock in the long

\(^{21}\)The instrument set used is detailed in the Notes to the Table. Our main results are highly robust to a range of alternative instrument sets
run, discussed in section (2.4.2). The change in uncertainty is negative and weakly significant at the 10% level, which is consistent with the predicted short run effect of higher uncertainty on investment for positive investment firms. More importantly the interaction term is negative and significant at the 5% level, which is again consistent with the prediction of our analysis in section (2.4.1). In Column (4) we drop the level uncertainty term and find that while the interaction term remains strongly significant, the change in uncertainty becomes insignificant. Column (5) reports our final preferred specification which includes our key term, the interaction between uncertainty and sales growth\textsuperscript{22}. This variable is negative and significant at the 5% level, consistent with our prediction that uncertainty reduces the firm's responsiveness to demand shocks\textsuperscript{23}.

Table (3.9) probes the results further to investigate where we are achieving identification of the uncertainty interaction term. We decompose the uncertainty measure into three components - a common macroeconomic factor ($\sigma_t$), a time-invariant firm specific factor ($\bar{\sigma}_i$) and a within-firm within-year residual component ($\tilde{\sigma}_u = \sigma_u - \bar{\sigma}_i - \bar{\sigma}_t$). Interestingly, it is the interactions with the firm specific components (especially $\tilde{\sigma}_u$) which are most informative. The aggregate uncertainty interaction is the least informative, being perversely signed in the first column and always insignificantly different from zero. This may explain why it has proved hard to identify significant effects

\textsuperscript{22} A joint test on the exclusion of the level and change in uncertainty terms from the specification in column (3) does not reject this restriction (p-value = 0.17).

\textsuperscript{23} Interestingly, this prediction that higher uncertainty reduces the response of investment to changes in demand also comes out from Nickell's (1978a) and (1978b) framework based on the effects of lags. Lags force the firm to be forward looking, with higher uncertainty leading to more gradual adjustment. Testing the distinction between these two models is the substance of future research.
of uncertainty using macroeconomic data. It is only by exploiting the micro
data that we observe sufficient variation in our measure of uncertainty to be
able to identify the effect of uncertainty in retarding the firm’s responsiveness
to shocks.
Table 3.8: Uncertainty and Investment

<table>
<thead>
<tr>
<th>Dependent Variable ((I_t/K_t-1))</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Growth ((\Delta y_t))</td>
<td>0.255</td>
<td>0.151</td>
<td>0.382</td>
<td>0.400</td>
<td>0.413</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.059)</td>
<td>(0.136)</td>
<td>(0.139)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>Change in Cash Flow ((\Delta C_t/K_{t-1}))</td>
<td>0.160</td>
<td>0.263</td>
<td>0.260</td>
<td>0.255</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.132)</td>
<td>(0.124)</td>
<td>(0.126)</td>
<td>(0.125)</td>
</tr>
<tr>
<td>Lagged Cash Flow ((C_{t-1}/K_{t-2}))</td>
<td>0.482</td>
<td>0.532</td>
<td>0.533</td>
<td>0.543</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.109)</td>
<td>(0.103)</td>
<td>(0.103)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>Error Correction Term ((y - k)_{t-1})</td>
<td>0.049</td>
<td>0.056</td>
<td>0.054</td>
<td>0.054</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.029)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Sales Growth Sqrd. ((\Delta y_t \times \Delta y_t))</td>
<td>0.481</td>
<td>0.512</td>
<td>0.494</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.152)</td>
<td>(0.150)</td>
<td>(0.151)</td>
<td></td>
</tr>
<tr>
<td>Change in Uncertainty ((\Delta \sigma_t))</td>
<td>-0.023</td>
<td>-0.012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.008)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Uncertainty ((\sigma_{t-1}))</td>
<td>-0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncert. x Sales Growth ((\sigma_t \times \Delta y_t))</td>
<td>-0.162</td>
<td>-0.165</td>
<td>-0.167</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.068)</td>
<td>(0.068)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Goodness of Fit - Corr. \((I/K, \hat{I}/\hat{K})^2\) | 0.259 | 0.287 | 0.285 | 0.285 | 0.307 |
| 2nd order serial correlation \((p)\) | 0.048 | 0.102 | 0.069 | 0.078 | 0.091 |
| Sargan \((p)\) | 0.510 | 0.709 | 0.699 | 0.629 | 0.560 |

NOTES: The total number of observations (for all columns) is 5347, on a sample period of 1973 to 1991, with 672 firms. A full set of time dummies is included in every specification. Estimation uses a GMM System estimator (see Blundell and Bond, 1998) calculated with DPD98 for Gauss (see Arellano and Bond, 1998). We report one step coefficient estimates with heteroskedasticity-consistent standard errors. The instruments used for columns (3) to (5) in the first-differenced equations are lags two and three of the variables: \((\frac{I}{K})_{t-2}\) and \((\frac{I}{K})_{t-3}\), \(\Delta y_{t-2}\) and \(\Delta y_{t-3}\), \((y - k)_{t-2}\) and \((y - k)_{t-3}\), \((\frac{C_{t-2}}{K_{t-3}})\) and \((\frac{C_{t-3}}{K_{t-4}})\), and \(\sigma_{t-2}\), \(\sigma_{t-3}\) and \(\sigma_{t-4}\); the instruments used in the levels equations are \(\Delta (\frac{I}{K})_{t-1}\), \(\Delta \Delta y_{t-1}\), \(\Delta (\frac{C_{t-1}}{K_{t-2}})\), \(\Delta \Delta (y - k)_{t-1}\) and \(\Delta \sigma_{t-1}\). Columns (1) and (2) use this instrument set but with the uncertainty variables excluded. Instrument validity is tested using a Sargan-Hansen test of the overidentifying restrictions for the two step GMM estimator. The test for no second order serial correlation in the first-differenced residuals is also reported.
Table 3.9: Separating Time, Firm and Residual Variation in Uncertainty.

<table>
<thead>
<tr>
<th>Dependent Variable ($I_t/K_{t-1}$)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Growth ($\Delta y_t$)</td>
<td>0.127</td>
<td>0.141</td>
<td>0.474</td>
<td>0.499</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.053)</td>
<td>(0.182)</td>
<td>(0.184)</td>
</tr>
<tr>
<td>Change in Cash Flow ($\Delta C_t/K_{t-1}$)</td>
<td>0.270</td>
<td>0.263</td>
<td>0.287</td>
<td>0.280</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.127)</td>
<td>(0.122)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>Lagged Cash Flow ($C_{t-1}/K_{t-2}$)</td>
<td>0.531</td>
<td>0.533</td>
<td>0.551</td>
<td>0.553</td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(0.103)</td>
<td>(0.100)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>Error Correction Term ($y - k)_{t-1}$</td>
<td>0.054</td>
<td>0.056</td>
<td>0.047</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Sales Growth Squared ($\Delta y_t \times \Delta y_t$)</td>
<td>0.497</td>
<td>0.507</td>
<td>0.534</td>
<td>0.537</td>
</tr>
<tr>
<td></td>
<td>(0.170)</td>
<td>(0.157)</td>
<td>(0.148)</td>
<td>(0.162)</td>
</tr>
<tr>
<td>Time Uncert. \times Sales Growth ($\sigma_t \times (\Delta y_t$)</td>
<td>0.016</td>
<td>-0.051</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.135)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Uncert. \times Sales Growth ($\sigma_t \times (\Delta y_t$)</td>
<td>-0.130</td>
<td>-0.136</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.107)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resid. Uncert. \times Sales Growth ($\sigma_{it} - \sigma_i - \sigma_{i-1} \times \Delta y_t$)</td>
<td>-0.225</td>
<td>-0.230</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.103)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goodness of Fit - Corr.$(I/K, I/K)$</td>
<td>0.307</td>
<td>0.298</td>
<td>0.311</td>
<td>0.288</td>
</tr>
<tr>
<td>2nd order serial correlation (p)</td>
<td>0.096</td>
<td>0.094</td>
<td>0.132</td>
<td>0.106</td>
</tr>
<tr>
<td>Sargan (p)</td>
<td>0.399</td>
<td>0.490</td>
<td>0.383</td>
<td>0.452</td>
</tr>
</tbody>
</table>

NOTES: The total number of observations (for all columns) is 5347, on a sample period of 1973 to 1991, with 672 firms. A full set of time dummies is included in every specification. Estimation uses a GMM System estimator (see Blundell and Bond, 1998) calculated with DPD98 for Gauss (see Arellano and Bond, 1998). We report one step coefficient estimates with heteroskedasticity-consistent standard errors. The instruments used in the first-differenced equations are lags two and three of the variables: $(\frac{K}{K})_{t-2}$ and $(\frac{K}{K})_{t-3}$, $\Delta y_{t-2}$ and $\Delta y_{t-3}$, $(y - k)_{t-2}$ and $(y - k)_{t-3}$, $(\frac{C_{t-2}}{K_{t-3}})$ and $(\frac{C_{t-3}}{K_{t-4}})$, and $\sigma_{t-2}$, $\sigma_{t-3}$ and $\sigma_{t-4}$; the instruments used in the levels equations are $\Delta (\frac{K}{K})_{t-1}$, $\Delta \Delta y_{t-1}$, $\Delta (\frac{C_{t-1}}{K_{t-2}})$, $\Delta \Delta (y - k)_{t-2}$ and $\Delta \sigma_{t-1}$. Instrument validity is tested using a Sargan-Hansen test of the overidentifying restrictions for the two step GMM estimator. The test for no second order serial correlation in the first-differenced residuals is also reported.

We have conducted many robustness tests on these results, some of which we now report. First, cash flow has no strong theoretical justification for being included in these models. Unfortunately, omitting the cash flow terms...
resulted in evidence of empirical mis-specification. Dropping both cash flow terms from the specification in column (5) of Table (3.8) caused the Sargan test to reject the overidentifying restrictions (p-value = 0.010) and produced significant second order serial correlation in the first-differenced residuals (p-value 0.049). Nevertheless, the interaction of uncertainty and sales growth was still found to be negative and significant in this specification, with a point estimate of -0.142 and a standard error of 0.065.

Secondly, we experimented with a range of additional non-linear and interaction terms, none of which were found to be statistically significant in our sample.\(^{24}\)

Thirdly, an implication of real options theory stressed by Guiso and Parigi (1999) is that the effect of uncertainty should be stronger for firms with more market power. We investigated whether the interaction term was stronger for firms in industries where market power is likely to be stronger (as proxied by concentration, trade barriers, etc.). We found no evidence that this was the case, although it could be that our industry-level proxies are not good measures of the firm’s market power.

Finally, we constructed an alternative measure of uncertainty after normalising the firm’s stock returns by the return on the FTSE All Share index for the same day. This measure gave somewhat more precise coefficient estimates than our basic results, presumably because some of the general stock market noise has been removed from the measure of uncertainty. For example, in the specification which corresponds to column (5) of Table (3.8), the

\(^{24}\)For example, we included interactions of uncertainty with squared sales growth, cash flow and the error correction term. The joint Wald test gave a \(\chi^2(3) = 4.42\) with a p-value of 0.219.
coefficient on the uncertainty interaction term rises to -0.196 with a standard error of 0.074. All these additional results are available from the authors on request.

### 3.6.1 Evaluation of the Quantitative Impact of Uncertainty on Investment Dynamics

The results suggest an important role for uncertainty in retarding the responsiveness of investment to demand shocks. We conducted some simple simulations where we increased sales in the firm permanently by 2.5%, 5% and 10%. We then tracked the path of investment and capital predicted by our preferred empirical specification as the firm responds to this shock. Our model suggests that firms with high uncertainty will respond more slowly to this shock than firms with low uncertainty. Consequently we examined the half life of the capital stock adjustment (how many years it takes the firm to get half-way towards its new long-run capital stock level) at different percentiles of the empirical uncertainty distribution (10th, 25th, 50th, 75th and 90th). Table (3.10) contains the results.

Table 3.10: Investment Half Lives in response to a 2.5%, 5% and 10% Shock to Demand, by Uncertainty Percentiles.

<table>
<thead>
<tr>
<th></th>
<th>10th ($\sigma=0.84$)</th>
<th>25th ($\sigma=1.08$)</th>
<th>50th ($\sigma=1.41$)</th>
<th>75th ($\sigma=1.89$)</th>
<th>90th ($\sigma=2.46$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>5%</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>10%</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

The exact size of the shock makes relatively little difference to the results. For the smallest shock, moving from the 25th to the 75th percentile of the uncertainty distribution increases the half life by two years. This is the
Figure 3.2: The Investment Response to a 2.5% demand shock for the 10th, 25th, 50th, 75th, and 90th percentiles of uncertainty.

Order of magnitude by which our measure of aggregate uncertainty increased between 1973 and 1975, a very large change by historical standards. This is illustrated in more detail in Figures (3.2) and (3.3), which track the paths of investment and the capital stock over a ten year period in response to a 2.5% demand shock.

The largest effects of different levels of uncertainty are manifest in the first year. So uncertainty is quantitatively important in retarding the investment response, but these effects are not large.
Figure 3.3: The Capital Stock after a 2.5% demand shock for the 10th, 25th, 50th, 75th, and 90th percentiles of uncertainty.

Notes: This simulates the firmlevel capital stock after a 2.5% demand shock using the parameters estimated in column (5) of Table 7 in the text. This response is plotted for the 10th, 25th, 50th, 75th and 90th percentiles of the distribution of our measure of uncertainty.
A second gauge of the importance of uncertainty and irreversibility is provided by comparing the gain in the goodness of fit of our preferred investment model in column (5) of Table (3.8), which includes the uncertainty interaction term and the squared sales growth term, in comparison to the more standard linear specification in column (1) of Table (3.8). To do this we calculate a year-by-year correlation between actual and predicted investment rates for both our preferred specification and the standard model. We take these two annual goodness of fit series and plot their annual difference in Figure (3.4) (left axis), as a time varying indicator of the improvement in fit from accounting for the effects of uncertainty and partial irreversibility on short run investment dynamics. In Figure (3.4) (right axis) we also plot the yearly average rate of change of sales growth as an indicator of the turning points in the business cycle. Turning points in the business cycle should be highlighted by rapid changes in the rate of sales growth as growth rates slow down going into recession or speed up heading into a boom.

It can be seen from Figure (3.4) that the improvement in fit tracks the positive turning points of the business cycles (correlation of 0.564), with large improvements evident in the late 1970s as the UK was recovering from the first oil shock, and again in the early 1980s when the UK was recovering from the monetarist experiment and the second oil shock. Interestingly this parallels the results of Caballero et al. (1995) and Cooper et al. (1999) who report that taking into account the non-linearities induced by fixed costs of investment leads to an improvement in investment fit most notably around turning points in the investment cycle. This suggests that accounting for

---

25The estimation approach of both papers differs from ours in that they estimate in-
Figure 3.4: Improvement in Fit From Adding an Uncertainty Interaction and Demand Squared Term (left axis), and Change of Growth Rate of Real Sales (right axis)

Notes: This graph plots on the left axis the year by year difference in correlation of actual investment rates with predicted investment rates using the uncertainty augmented model (column 6 of Table 7) and predicted investment rates using the standard model (column 1 of Table 7). Positive values represent an improvement in fit from using the uncertainty interaction term. Plotted on the right axis is the yearly average change of sales growth as an indicator of the business cycles turning points.
the non-linearities induced by partial irreversibility will be most important in periods of large investment fluctuations.

3.7 Conclusions

In this paper we have presented a theoretical framework for analysing firm-level investment under uncertainty. We characterise the problem as one where a firm has multiple projects ('plants' or 'lines of capital'). Under fairly general conditions over the production function and the distribution of the demand shocks facing the firm we approximate the aggregate firm-level investment dynamics to a second order. We emphasise a neglected theoretical implication of real options theory. Firms facing increased uncertainty should be more cautious - they should exhibit a lower responsiveness of investment to demand shocks than firms subject to less uncertainty. This approach also predicts a non-linear response of firm investment to demand shocks.

We test these predictions using a panel of 672 UK manufacturing firms between 1973 and 1991. Using a measure of uncertainty based on the daily stock market returns of our firms, we estimate non-linear error correction models of investment. The key implication of the theory is supported by the data, with a significantly lower response of investment to sales growth when uncertainty is high. This is robust to a number of experiments and primarily related to the firm-specific components of uncertainty rather than to general macroeconomic uncertainty. This may explain why it is difficult to detect these important effects in the macro data. We also find significant investment models at the plant level and track the cross sectional distribution of plant level investment to model aggregate investment.
non-linearity in the investment dynamics, with a convex relationship between investment rates and sales growth, as predicted by our theoretical analysis. As a secondary result, we find no significant evidence of any effect of our uncertainty measures on the level of the capital stock in the long run. This finding is also quite consistent with our theoretical framework.

This work is of course only a first step. The implications for short run investment dynamics also apply to more broadly defined investment goods, such as R&D and the development of information technology, which are clearly subject to considerable irreversibilities and uncertainty. More broadly, the hiring and training of labour could also be regarded as an investment process and would naturally be included as another type of 'capital' good in our general class of production functions. Investigating the importance of uncertainty for employment adjustment and the interrelationship between threshold behaviour for different inputs should be high on the agenda of future research.
3.8 Appendices

Appendix A: Further Proofs

Proof of Proposition 1

Consider a firm which is thinking about undertaking a marginal investment. One way of formulating its decision is in terms of its choice over investing a unit of capital today versus waiting until next period. Let the firm’s one period marginal revenue of capital be denoted by \( R_K(K, P) \Delta t \) where \( \Delta t \) is the length of the period, \( K \) is the current capital stock, \( P \) is the current level of market demand, and \( B \) is the price of a unit of capital. Let the firm’s value function be denoted \( V(K, P) \), and its one period discount rate be denoted \( \gamma = \exp(-r\Delta t) \). We examine the impact of a change in the distribution of demand shocks in the current period on its investment thresholds in the current period\(^{26}\). Since all future demand distributions are held constant the functional form of next period’s continuation value function will be unaffected. This allows us to use the Bellman equation to examine the impact of temporary changes in demand distributions on the current investment thresholds while holding the future investment policy constant.

If the firm’s optimisation problem satisfies assumptions (1) to (3) in section (3.3.2), then from Eberly and Van Mieghem (1997) the optimal investment strategy for each line of capital can be characterised by the investment and disinvestment demand triggers \( U \) and \( L \). For a level of demand just below the current investment threshold \( U^* \), which is a function of the distrib-

\(^{26}\)The timing assumptions are that the firm invests then experiences a demand shock each period.
bution of current demand shocks, we can model the return from a marginal investment for each line of capital in terms of investing this period versus investing next period:

\[
\text{Investment This Period: } R_K(K, P)\Delta t - B + \gamma \int_{-\infty}^{\infty} V_K(K, P)dF(P) \tag{3.12}
\]

\[
\text{Investment Next Period: } \gamma \int_{U}^{\infty} (V_K(K, P) - B)dF(P) \tag{3.13}
\]

where the integral is taken with respect to \( F(P) \), the cumulative probability distribution of current demand shocks. At the investment margin \( U^* \) these two returns will be equal and we can combine them to write:

\[
R_K(K, U^*)\Delta t - B(1 - \gamma) - \gamma \int_{-\infty}^{U} (B - V_K(K, P))dF(P) = 0 \tag{3.14}
\]

This first two terms can be interpreted as the marginal returns to investment in the current period, \( R_K(K, U^*)\Delta t \), less the cost of capital \( B(1 - \gamma) \) on paying for investment this period rather than next period. The last term, \( \gamma \int_{-\infty}^{U} (B - V_K(K, P))dF(P) \), is equal to the value of a put option on a marginal investment next period with a strike price of \( B \). Since the value of this put option must be positive, and the marginal revenue product is concave, the investment threshold this period will be above its reversible level.

Similarly we can also frame the marginal disinvestment decision at the current lower trigger \( L^* \) in terms of disinvesting this period versus for a capital resale price of \( S \) versus waiting until next period

\[
\text{Disinvestment This Period: } S \tag{3.15}
\]

\[
\text{Disinvestment Next Period: } R_K(K, P)\Delta t + \gamma \int_{-\infty}^{L} SdF(P) + \gamma \int_{L}^{\infty} V_K(K, P)dF(P) \tag{3.16}
\]
At this period's disinvestment margin $L^*$ these two returns will be equal and we can write

$$R_K(K, L^*) \Delta t - S(1 - \gamma) + \gamma \int_{L}^{\infty} (V_K(K, P) - S) dF(P) = 0$$

(3.17)

This firm two terms can be interpreted as the one period marginal returns to delaying the disinvestment decision one period, whilst the last term, $\gamma \int_{L}^{\infty} (V_K(K, P) - S) dF(P)$, is equal to the value of a call option on a marginal disinvestment next period with a strike price of $S$. Since the value of this call option must be positive the disinvestment threshold will be below its reversible level.

In proceeding we will use the following notation $G(P) = \int_{-\infty}^{P} (F_B(X) - F_A(X)) dX$ to simplify exposition. It can be shown that an equivalent definition of second order stochastic dominance of $F_A$ over $F_B$ is that $G(P) \geq 0 \forall P$. The following Lemma will prove useful

**Lemma 1:**

$$\int_{-\infty}^{\infty} V_K(K, P)(dF_B(P) - dF_A(P)) = \int_{L}^{U} V_{KPP}(K, P)G(P)dP$$

**Proof:**

$$\int_{-\infty}^{\infty} V_K(K, P)(dF_B(P) - dF_A(P))$$

$$= [V_K(K, P)(F_B(P) - F_A(P))]_{-\infty}^{\infty}$$

$$- \int_{-\infty}^{\infty} V_{KP}(K, P)(F_B(P) - F_A(P))dP$$

$$= - \int_{-\infty}^{\infty} V_{KP}(K, P)(F_B(P) - F_A(P))dP$$

$$= - [V_{KP}(K, P)G(P)]_{-\infty}^{\infty}$$

---

27See, for example, Rothschild and Stiglitz (1970).
\[ + \int_{-\infty}^{\infty} V_{KP}(K, P)G(P)dP \]
\[ = \int_{L}^{U} V_{KP}(K, P)G(P)dP \]

The first and third lines follow by Riemann-Stieltjes integration by parts, and the second and fourth lines follow by the fact that:

\[
\begin{align*}
V_K(K, P) &= B & V_{KP}(K, P) &= 0 \quad \text{if } P > U \\
S \leq V_K(K, P) \leq B & \quad \text{and} \quad V_{KP}(K, P) > 0 \quad \text{if } L \leq P \leq U \\
V_K(K, P) &= S & V_{KP}(K, P) &= 0 \quad \text{if } L < P
\end{align*}
\]

so that \(V_{KP}(K, P)\) and \(V_{KP}(K, P)\) are zero outside the region of inaction.

At the investment threshold we take a total difference of (3.14) allowing the distribution of this period’s demand shocks to change and this period’s investment threshold to change but holding everything else from next period on constant. This allows us to examine the impact of a short run change in uncertainty on the current investment thresholds. This total difference can be written as,

\[ R_{KP}(K, U)dU^* - \gamma \int_{-\infty}^{U} (B - V_K(K, P))(dF_B(P) - dF_A(P)) = 0 \]

The second term is positive for a distribution \(F_B\) which is second order stochastically dominated by \(F_A\), but which has equal or lesser expected marginal value (i.e. that is \(\int_{-\infty}^{\infty} V_K(K, P)(dF_B(P) - dF_A(P)) \leq 0\)). To see this expand this second term using integration by parts

\[
\int_{-\infty}^{U} (B - V_K(K, P))(dF_B(P) - dF_A(P)) = [(B - V_K(K, P))(F_B(P) - F_A(P))]_{-\infty}^{U}
\]

\[^{28}\text{See the characterisation of the optimal investment strategy in section (3.3.2) in the main body of the paper or in Eberly and Van Mieghem (1997).}\]
\[ + \int_{-\infty}^{U} V_{KP}(K, P) (F_{B}(P) - F_{A}(P)) dP \]
\[ = \int_{-\infty}^{U} V_{KP}(K, P) (F_{B}(P) - F_{A}(P)) dP \]
\[ = [V_{KP}(K, P))G(P)]_{-\infty}^{U} \]
\[ - \int_{-\infty}^{U} V_{KP}(K, P)G(P)dP \]
\[ = V_{KP}(K, U)G(U) \]
\[ - \int_{L}^{U} V_{KP}(K, P)G(P)dP \]
\[ \geq 0 \text{ if } \int_{-\infty}^{\infty} V_{K}(K, P)(dF_{B}(P) - dF_{A}(P)) \leq 0 \]

The first and third lines follow by Riemann-Stieltjes integration by parts; the second line follows because \( F_{A}(\infty) = F_{B}(\infty) = 0 \) and \( V_{K}(K, U) = 0 \); the fourth line follows because \( \int_{-\infty}^{\infty} (dF_{B}(X) - dF_{A}(X)) dX = 0 \) and \( V_{KP}(K, P) = 0 \) if \( P < L \); while the fifth line follows because \( G(U) \geq 0 \) by stochastic dominance of \( F_{A} \) over \( F_{B} \) and by Lemma 1.

Hence, we can write that
\[ dU^* = \gamma \frac{\int_{L}^{U} V_{KP}(K, P)G(P)dP}{R_{KP}(K, U^*)} \]
\[ \geq 0 \text{ if } \int_{-\infty}^{\infty} V_{K}(K, P)(dF_{B}(P) - dF_{A}(P)) \leq 0 \]

At the disinvestment threshold we take a total difference of (3.17) allowing the distribution of future demand shocks and the disinvestment threshold to change, and by similar arguments we can derive
\[ dL^* = \frac{\gamma}{R_{KP}(K, L^*)} \left( -V_{KP}(K, L))G(L) + \int_{L}^{U} V_{KP}(K, P)G(P) \right) \]
\[ \leq 0 \text{ if } \int_{-\infty}^{\infty} V_{K}(K, P)(dF_{B}(P) - dF_{A}(P)) \geq 0 \]

So that for a change in distribution from \( F_{A} \) to \( F_{B} \) whereby the latter is stochastically dominated by the former but they both yield the same expected
marginal value of capital, so that \( \int_{-\infty}^{\infty} V_K(K, P)(dF_B(P) - dF_A(P)) = 0 \),
the investment threshold will move up and the disinvestment threshold will
move down. Combining these two conditions (3.18) and (3.19) and imposing
condition (3) from section (3.3.2), we find that \( R_{KP}(K, U^*) = R_{KP}(K, L^*) \)
by the multiplicative nature of the demand shock in the revenue function, so
that
\[
dU^* - dL^* = \frac{\gamma}{R_{KP}(K, U^*)} \left( \begin{array}{c} V_{KP}(K, U)G(U) \\ -V_{KP}(K, L)G(L) \end{array} \right)
\geq 0
\]
for any \( F_B \) which is stochastic dominated by \( F_A \) regardless of whether \( \int_{-\infty}^{\infty} V_K(K, P)(dF_B(P) - dF_A(P)) \) is positive or negative. Since this holds without loss of generality
for every line of capital in a multi-line of capital revenue function we obtain
the necessary result.

\[\Box\]

**Proof of Proposition 2**

With costless reversibility the firm's revenue \( R(K_1, K_2, \ldots K_N, P) \) is equal
to current sales, \( Y(K_1, K_2, \ldots K_N, P) \), less capital costs \( \sum_i (r + \delta)_i K_i \)
\[
R(K_1, K_2, \ldots K_N, P) = Y(K_1, K_2, \ldots K_N, P) - \sum_i (r + \delta)_i K_i \quad (3.20)
\]
The first order conditions for profit maximisation are
\[
\partial Y(K_1, K_2, \ldots K_N, P)/\partial K_i = (r + \delta)_i \quad i = 1, 2, \ldots N \quad (3.21)
\]
Using the property that \( Y(K_1, K_2, \ldots K_N, P) \) is jointly homogeneous of degree
one in \( (K, P^{1-\lambda}) \), so that \( \partial Y(K_1, K_2, \ldots K_N, P)/\partial K_i \) is jointly homogeneous of
degree zero in \((K, P^{\frac{1}{1-\lambda}})}\), these can be restated in matrix form as

\[
\begin{align*}
\frac{\partial Y}{\partial K_1} (K_1 P^{-\frac{1}{1-\lambda}}, K_2 P^{-\frac{1}{1-\lambda}}, \ldots, K_N P^{-\frac{1}{1-\lambda}}, 1) / \partial K_1 &= (r + \delta)_1 \\
\frac{\partial Y}{\partial K_2} (K_1 P^{-\frac{1}{1-\lambda}}, K_2 P^{-\frac{1}{1-\lambda}}, \ldots, K_N P^{-\frac{1}{1-\lambda}}, 1) / \partial K_2 &= (r + \delta)_2 \\
& \vdots \quad (3.22) \\
\frac{\partial Y}{\partial K_N} (K_1 P^{-\frac{1}{1-\lambda}}, K_2 P^{-\frac{1}{1-\lambda}}, \ldots, K_N P^{-\frac{1}{1-\lambda}}, 1) / \partial K_N &= (r + \delta)_N
\end{align*}
\]

which represents \(N\) equations for \(\partial Y/\partial K_1, \partial Y/\partial K_2, \ldots, \partial Y/\partial K_N\) in the \(N\) unknowns \(K_1 P^{-\frac{1}{1-\lambda}}, K_2 P^{-\frac{1}{1-\lambda}}, \ldots, K_N P^{-\frac{1}{1-\lambda}}\). Since \(Y(K_1, K_2, \ldots, K_N, P)\) is continuously differentiable it is a continuously differentiable mapping from \(\mathbb{R}^N \rightarrow \mathbb{R}^1\). Furthermore, \(Y(K_1, K_2, \ldots, K_N, P)\) is strictly concave so that the matrix of second derivatives with respect to capital \(Y_{KK}(K_1, K_2, \ldots, K_N, P) : \mathbb{R}^N \rightarrow \mathbb{R}^N\)

will have full rank. Hence, we can use the inverse function theorem to write

\[
\begin{align*}
K_1 P^{-\frac{1}{1-\lambda}} &= h_1((r + \delta)_1, (r + \delta)_2, \ldots, (r + \delta)_N) \\
K_2 P^{-\frac{1}{1-\lambda}} &= h_2((r + \delta)_1, (r + \delta)_2, \ldots, (r + \delta)_N) \\
& \quad \vdots \quad (3.23) \\
K_N P^{-\frac{1}{1-\lambda}} &= h_N((r + \delta)_1, (r + \delta)_2, \ldots, (r + \delta)_N)
\end{align*}
\]

Defining \(A_i = h_i((r + \delta)_1, (r + \delta)_2, \ldots, (r + \delta)_N)\) this can be re-written as

\[
K_i = A_i P^{\frac{1}{1-\lambda}}, \quad i = 1, 2, \ldots, N \quad \text{some } A_i \in \mathbb{R}^1 \quad (3.24)
\]

so that \(\sum K_i = P^{\frac{1}{1-\lambda}} \sum A_i\), which can be stated in logs as,

\[
\log(\sum K_i) = \log(\sum A_i) + \frac{1}{1-\lambda} \log P \quad (3.25)
\]

Using the result (3.24) in writing out the sales function we obtain

\[
Y(K_1, K_2, \ldots, K_N, P) = Y(A_1 P^{\frac{1}{1-\lambda}}, A_2 P^{\frac{1}{1-\lambda}}, \ldots, A_N P^{\frac{1}{1-\lambda}}, P) \quad (3.26)
\]

\[
= P^{\frac{1}{1-\lambda}} Y(A_1, A_2, \ldots, A_N, 1) \quad (3.27)
\]

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where the second line follows by homogeneity. Hence, taking logs and using (3.25) to substitute out \( \frac{1}{1-\lambda} \log P \) delivers

\[
\log(\sum K_i) = \log \frac{(\sum A_i)}{Y(A_1, A_2, \ldots A_N, 1)} + \log Y(K_1, K_2, \ldots K_N, P) \tag{3.28}
\]

Appendix B: Data

The UK data is taken from the published accounts of manufacturing firms listed on the UK stock market. We deleted firms with less than three consecutive observations, broke the series for firms whose accounting period fell outside 300 to 400 days due to changes in year end timing, and excluded the observations for firms where there are jumps of greater than 150% in any of the variables. This data is obtained from the Datastream on-line service.

Investment \((I)\). Total new fixed assets less fixed asset sales: DS435-DS423.

Capital Stock \((K)\): Constructed by applying a perpetual inventory procedure with a depreciation rate of 8%. The starting value was based on the net book value of tangible fixed capital assets in the first observation within our sample period, adjusted for previous years inflation. Subsequent values were obtained using accounts data on investment and asset sales, and an aggregate series for investment goods prices.

Sales \((Y)\): Total sales, DS104, deflated by the aggregate GDP deflator.

Cash Flow \((C)\): Net profits (earned for ordinary), DS182, plus depreciation, DS136.

Uncertainty \((\sigma)\). The computation of this variable is described in the text. For a company we take the daily stock market return (Datastream Returns
Index, RI). This measure includes on a daily returns basis the capital gain on the stock, dividend payments, the value of rights issues, special dividends, and stock dilutions. We then compute the standard deviation of these daily returns on a year by year basis matched precisely to the accounting year, and adjust for the firm’s debt-equity ratio as in Leahy and Whited (1998). We trim the variable so that values above five are set equal to five. The results are robust to dropping these ten observations.

Appendix C: Estimation with Simulated Investment Data

The simulated data is based on the canonical investment model outlined in section (3.3). It is generalised to allow for partially irreversible labour as well as capital. It incorporates aggregation by assuming that the firms operate a number of production plants which experience both idiosyncratic plant level shocks and common firm level shocks. This procedure generates lumpy plant-level investment and employment data and smoother firm-level investment and employment data. For the US Compustat data base, this simulated firm-level data appears to closely parallel the main time series and cross sectional properties of actual firm-level data (see Bloom 2000b). This simulation method is outlined below.

Plants are assumed to have a Cobb-Douglas revenue function, $PK^aL^b$, where $P$ is a demand term, $K$ is capital, $L$ is labour and $0 < a + b < 1$. Both factors are assumed to be partially irreversible. Demand is stochastic with $P$ evolving as a Brownian motion process. This Brownian motion process can be broken down into two Brownian sub-components. The first is an observable
firm-level shock which is common to all plants. The second is an idiosyncratic
plant-level shock which is independent across plants and has a zero mean. These plant-level shocks are assumed to be unobservable in our firm-level
data set.

For this framework Eberly and Van Mieghem (1997) prove that the optimal investment and employment thresholds for each plant can be modelled according to the threshold rules laid out in Table (3.11). Investment only occurs when the marginal revenue product of capital, \( aP K^{a-1}L^b \), is equal to its investment user cost of capital, \( b \), times an investment real options term, \( \phi^K_I > 1 \). And disinvestment only occurs when the marginal revenue product falls to its disinvestment cost of capital, \( s \), divided by a disinvestment real options term, \( \phi^K_D > 1 \). In between these thresholds the plant will undertake no investment or disinvestment. We also consider the adjustment process for labour, which like capital, will be partially irreversible due to hiring, training and firing costs. Thus, for labour we can characterise the hiring and firing thresholds in terms of the marginal revenue product of labour, \( bAK^aL^{b-1} \), the present discounted costs of hiring one worker, \( h \), the present discounted savings from firing one worker, \( f \), and the hiring and firing real options terms, \( \phi^L_H > 1 \) and \( \phi^L_F > 1 \) respectively.

These real options terms can be numerically calculated given our assumptions on the parameters in the firm's environment - the discount rate (10%), the revenue function parameters (\( a = 0.25 \) and \( b = 0.5 \)), the ratio of \( b/s = 0.5 \).

---

29 The plant level shock can always be defined to have a zero mean through the definition of the firm level shock as the average plant level shock in each period. The assumption that the plant level shocks are independent across plans can be weakened to imperfect correlation across plants, and is made for the simplicity of the simulation exercise.

30 Where \( h > f > 0 \) because hiring and firing both involve costs which are not recouped.
Table 3.11: The Simulation Thresholds for Investment and Hiring.

<table>
<thead>
<tr>
<th>Action</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest if</td>
<td>$aAK^{a-1}L^b \geq b \times \phi^K$</td>
</tr>
<tr>
<td>Hire if</td>
<td>$bAK^aL^{b-1} \geq h \times \phi^K$</td>
</tr>
<tr>
<td>Do Nothing</td>
<td>$s/\phi^K_L &lt; aAK^{a-1}L^b &lt; b \times \phi^K$</td>
</tr>
<tr>
<td>Do Nothing</td>
<td>$f/\phi^K_F &lt; bAK^aL^{b-1} &lt; h \times \phi^K_H$</td>
</tr>
<tr>
<td>Disinvest if</td>
<td>$aAK^{a-1}L^b \leq s/\phi^K_D$</td>
</tr>
<tr>
<td>Fire if</td>
<td>$bAK^aL^{b-1} \leq f/\phi^K_F$</td>
</tr>
</tbody>
</table>

and $h/f = 0.9$. These parameters are chosen to match survey evidence on production functions, the cost of capital (see, for example, Summers (1987)) and the low rates of disinvestment at the firm level. The labour hiring/firing prices were chosen as economically reasonable values (see, for example, Nickell (1986)) which also provide a good fit between actual and simulated data.

Finally, we proxy the firm specific degree of uncertainty by using the mean standard deviation of the firm’s daily share returns, $\sigma_i$, over our sample period$^{31}$. This measure of uncertainty is here defined so as to be time invariant in line with Eberly and Van Mieghem’s (1997) modelling assumptions.

The firm-level demand shock is proxied for by the growth in real firm sales. For each firm we also need to start with an initial distribution of projects between these investment, disinvestment, hiring and firing thresholds. This is generated by starting all projects as identical and running the simulation for twenty years to obtain an empirical ergodic distribution, which is then assumed to be the initial distribution we use to generate our simulated data$^{32}$. Our simulated firm level investment and hiring data is then calculated by adding up across all plants within each firm. In these simulations we assumed

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$^{31}$These values are divided by 5 here to re-scale them in line with the magnitudes of annual sales and price variation.

$^{32}$Not surprisingly, this assumption leads to violations of the initial conditions restrictions required by the System GMM estimator we use in our main empirical analysis. For this reason we present results for the simulated data using only equations in first differences, for which lagged levels of the variables are used as instruments.
all firms operate 20 plants\textsuperscript{33}.

In Table (3.12) we present the results from GMM estimation of error correction models using the simulated capital stock and investment data, \(K^S\) and \(\left(\frac{I^S}{K^S}\right)\) respectively, and the actual sales data used to generate this. The sample is exactly the same as that used to estimate our results in Tables (3.8) and (3.9) in the main text.

Table 3.12: GMM Estimates Using The Simulated Investment Data.

<table>
<thead>
<tr>
<th>Dependent Variable ((I^S_t/K^S_{t-1}))</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Growth ((\Delta y_{it}))</td>
<td>0.501</td>
<td>0.412</td>
<td>0.317</td>
<td>0.461</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Error Correction Term ((y - k^S)_{i,t-1})</td>
<td>0.274</td>
<td>0.263</td>
<td>0.266</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.021)</td>
<td>(0.011)</td>
<td></td>
</tr>
<tr>
<td>Sales Growth Sqr. ((\Delta y_{it} \times \Delta y_{it}))</td>
<td>0.453</td>
<td>0.446</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.060)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncert. \times Sales Growth ((\sigma_i \times \Delta y_{it}))</td>
<td>-0.085</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd order serial correlation ((p))</td>
<td>0.000</td>
<td>0.758</td>
<td>0.223</td>
<td>0.231</td>
</tr>
<tr>
<td>Sargan ((p))</td>
<td>0.000</td>
<td>0.252</td>
<td>0.127</td>
<td>0.214</td>
</tr>
</tbody>
</table>

NOTES: - The total number of observations (for all columns) is 5347, on a sample period of 1973 to 1991, with 672 firms. A full set of time dummies is included in every specification. Estimation uses a first-differenced GMM estimator computed in DPD98 for Gauss (see Arellano and Bond, 1998). One step coefficients and heteroskedasticity-consistent standard errors are reported. The instruments used for all equations are lags two and three of the variables: \(\left(\frac{I^S}{K^S}\right)_{i,t-2}\) and \(\left(\frac{I^S}{K^S}\right)_{i,t-3}\), \(\Delta y_{i,t-2}\) and \(\Delta y_{i,t-3}\), \((y - k)_{i,t-2}\) and \((y - k)_{i,t-3}\), and \(\sigma_i\). Instrument validity is tested using a Sargan-Hansen test of the overidentifying restrictions for the two step GMM estimator. The test for no second order serial correlation in the first-differenced residuals is also reported.

The high degree of residual autocorrelation in the static accelerator spec-

\textsuperscript{33}Perhaps surprisingly, the number of plants we assume that operate within each firm appears to make very little difference to our panel estimation results. The simulated results derived assuming firms operate 2 or 200 plants look almost identical to those presented in Table AII using 20 plants.
ification of column (1) and the failure of the Sargan test is testament to the strong dynamics in this simulated data arising from the lagged effect of demand shocks on investment though ‘pent up demand’. In column (2) we include a lagged error correction term which reflects the deviation of the firm’s capital/output ratio from its long run level. This removes the finding of significant second order serial correlation in the first-differenced residuals, and the Sargan test does not reject this specification. In column (3) we add in the squared sales growth term, which is found to be positive and highly significant. Our test thus correctly rejects the null hypothesis of a linear relationship between investment rates and sales growth, and the convex relationship detected is that predicted in section (3.3.4).

Finally, in column (4) we also include the interaction term between sales growth and uncertainty. This interaction term is found to be negative and significant at the 5% level. Again our empirical test correctly rejects the null hypothesis of a common response to demand shocks for high uncertainty and low uncertainty firms, and we detect the predicted weaker response of investment to demand shocks at higher levels of uncertainty.

We conclude that if the data were generated by a partial irreversibility model, our empirical tests should be able to reject the linear error correction specification, and the inclusion of quadratic and interaction terms should detect the correct signs on these additional variables.
Chapter 4

The Dynamic Demand for Capital and Labour: Modelling Real Options and Irreversibility

4.1 Abstract

This paper studies the effects of real options and irreversibilities on firms’ joint investment and employment decisions. A simulation model is constructed and its structural parameters estimated by indirect inference on US firm level data. The predicted time series and cross sectional responses of the model are shown to closely mimic those in the data set. Given the structural nature of this estimator it should provide a useful tool for modelling the effects of policy interventions on firms’ investment and employment decisions under different degrees of uncertainty.
4.2 Introduction

The standard approach to modelling investment and labour demand under uncertainty considers a firm operating a single production process and using a homogeneous capital good\(^1\). Investment and hiring decisions are assumed to be (partially) irreversible and market demand uncertain. This generates real options on these decisions and a separation of the thresholds for investment/disinvestment and hiring/firing, with no action undertaken in between. Even low levels of uncertainty and irreversibility can lead these thresholds to be significantly spaced apart in relation to their positions under complete certainty and costless reversibility. This changes the optimal investment and hiring behaviour of firms from being smooth and continuous to one that is lumpy and frequently zero.

At first sight firm-level investment and labour demand series appear too smooth to be consistent with these models. But in micro establishment-level data, like the US Longitudinal Research Database (LRD) and the UK Annual Respondents’ Database (ARD), such lumpy investment and hiring with frequent zeros is observed, particularly for smaller plants\(^2\). This suggests that observations with zero investment and continuous hiring at the firm level occur infrequently simply because of aggregation across multiple plants. This is not surprising - firms are often observed to operate multiple production lines, plants and subsidiaries, each employing many types of capital goods and types of labour. If these processes are not perfectly correlated

\(^1\)See Bertola (1988), Pindyck(1988) and Dixit and Pindyck (1994), for example.
due to idiosyncratic shocks and heterogeneous technologies then aggregation will smooth away much of the lumpiness from a firm-level series. Nevertheless uncertainty will still play an important role in determining firm-level investment and hiring through its effects on the decisions for the individual production plants. This has been shown in a number of papers on macro investment and consumption which demonstrate that aggregation does not diminish the effects of lumpy micro level behaviour.

In this paper I go beyond previous studies of irreversibilities by jointly modelling investment and labour demand. This is undertaken in a framework which allows for the kind of small numbers aggregation which is commonly observed in firm level data. This is a two stage modelling process. First, I model the firm as operating a number of separate production projects. Each project is assumed to have a Cobb-Douglas capital and labour production function and be subject to Brownian motion demand shocks. This allows me to explicitly solve their optimal investment and hiring strategies. Second, these projects are assumed to face a common firm level shock taken from the firm level sales data and an idiosyncratic project level shock. This project level shock is simulated and the project level investment and hiring responses aggregated up to the firm level.

Using this simulation approach I can then develop a structural estimator of joint firm level hiring and investment decisions. I use indirect inference

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3 See, for example, Caballero (1993) and Eberly (1994) on aggregation across consumer durables, and Bertola and Caballero (1994), Caballero and Engel (1999), Cooper et al. (2000) and Attanasio et al. (2000) on aggregation across plant level investment.

4 Projects are defined theoretically as a single output good production process. This can correspond to small production plants. In the UK ARD plant level data set however, production plants often turn out to produce several products, so that they represent an aggregation across production projects.
to uncover the free structural parameters. The predicted investment and employment responses using these parameters are shown to closely mimic the cross sectional and time series behaviour of actual investment and employment in my Compustat panel of US firms. The structural nature of this approach allows me to use it to forecast the effects of policy interventions, such as interest rate or tax changes, on investment and employment.

The plan of the paper is as follows. Section 2 introduces the model for a single project firm operating partially irreversible capital and labour. This is then extended to allow for a multi-product firm and the effects of aggregation. Section 3 then discusses the estimation and simulation procedures necessary to identify the underlying free structural parameters in the model. Section 4 describes the Compustat firm level data set used in this estimation, while section 5 discusses the estimation results. Some concluding remarks are made in section 6.

4.3 A Model of Investment and labour Demand

4.3.1 The Single Project Firm

I assume that firm has the following revenue function $R(K, L, P)$ in terms of its capital stock ($K$), the labour force ($L$) and demand conditions ($P$),

$$R(K, L, P) = AP^\gamma K^{\alpha(1-\gamma)} L^{(1-\alpha)(1-\gamma)}$$  \hspace{1cm} (4.1)

This revenue function can be shown to nest, for example, a Cobb-Douglas production function and an iso-elastic demand curve. I also assume that the demand conditions follow a Brownian motion process with drift $\mu$ and
variance $\sigma^2$. The firm is assumed to maximize the expected present value of revenues minus the cost of buying capital at a price $B$, plus the proceeds received from selling capital at a price $S$, plus the costs of hiring workers at cost $E$ less the cost of firing workers at cost $F$. The optimal investment and hiring policy then solves the firm's profit maximization problem

$$
\max_{\{I(s)\}} \left\{ \int_t^\infty \exp^{-r(s-t)} \left( R(K, L, P) - BdI^+(s) + SdI^-(s) \right) - EdH^+(s) - FdH^-(s) \right\} \tag{4.2}
$$

subject to

$$
dK(t) = -\delta_K K dt + I^+(t) - I^-(t)
$$

$$
dL(t) = -\delta_L L dt + H^+(s) - H^-(s)
$$

where $r$ is the discount rate, $I^+$ and $I^-$ denote positive and negative investment and $H^+$ and $H^-$ denote hiring and firing.

The investment and hiring solution to the that maximizes the firm's profits has been shown by Eberly and Van Mieghern (1997) to have the form outlined in Table (4.1), where $R_K(K, L, P)$ and $R_L(K, L, P)$ represent the marginal revenue product of labour and capital respectively and $B^* \geq S^*$ and $E^* \geq F^*$.

**Table 4.1: The Threshold Behavior for Investment and Hiring.**

| Invest if: | $R_K(K, L, P) \geq B^*$ | Hire if: | $R_L(K, L, P) \geq E^*$ |
| Do Nothing if: | $S^* < R_K(K, L, P) < B^*$ | Do Nothing if: | $F^* < R_L(K, L, P) < E^*$ |
| Dis-Invest if: | $R_K(K, L, P) \leq S^*$ | Fire if: | $R_L(K, L, P) \leq F^*$ |

Because the revenue function (4.1) has a log-linear form these investment, disinvestment, hiring and firing thresholds also have a log-linear form and are plotted in Figure (4.1).

Figure (4.1) plots on the x-axis the difference between logged demand and logged employment, which is a measure of labours marginal revenue product.
Figure 4.1: The Central Region of Inaction, and the Investment, Disinvestment, Hiring & Firing Thresholds
of labour. On the y-axis the difference between logged demand and logged capital is plotted, which is a measure of the marginal revenue product of capital. In Figure (4.1) I have assumed that labour is cheaper to adjust than capital, and so the hiring and firing thresholds are much closer to each other than the investing and dis-investing thresholds. In between these thresholds there is a region of inaction in which the firm will do nothing to change its labour force or capital stock.

4.3.2 The Multi Project Firm

While this may be an appropriate characterization of hiring and investment at the project level the data sources that are available to me at the establishment and firm level data reveals that aggregation across a number of production projects is common\(^5\). To account for this pervasive phenomenon of aggregation I model the firm as a collection individual production projects and processes. To obtain an analytically tractable solution for the firms investment and hiring response I assume that the firms revenue function is linearly separable across its production projects, so that it has the following form\(^6\)

\[
R_f(K_1, K_2, \ldots K_N; L_1, L_2, \ldots L_N; P_1, P_2, \ldots P_N; P_f) = P_f^{\frac{N}{m}} \sum_{i=1}^{N} R(K_i, L_i, P_i) \tag{4.3}
\]

\(^5\)See, for example, the discussion in Doms and Dunne (1998), Attanasio et al (2000), and Bloom, Bond and Van Reenen (2001).

\(^6\)Actually only the marginal revenue product needs to be separable so that the revenue product can be non-separable in the existence of different projects. An example of this difference would be a firm which derived positive value from combining upstream and downstream production processes, but with this value creation being independent of the size of either process. These would then have dependent revenue products but independent marginal revenue products.
\[ P_f^T \sum_{i=1}^{N} A_i P_i^T K_i^{a(1-\theta)} L_i^{(1-\sigma)(1-\theta)} \]

where \( K_i, L_i \) and \( P_i \) are the project level capital stock, labour force and demand term, and \( P_f \) is the firm level demand term common to all projects in the firm. This common firm level sales shock \( P_f \) and all the idiosyncratic project level shocks \( P_1, P_2, \ldots P_N \) are also assumed to be independent Geometric Brownian motion processes\(^7\). The firm and project level shocks are assumed to have (mean, variance) of \((\sigma_F^2, \mu)\) and \((\sigma_P^2, 0)\) respectively.

While it would be desirable to generalize this by introducing interactions between individual projects within the firm this is not analytically tractable. The difficulty is that the distribution of projects would become a state variable in the firm’s optimization problem and this would dramatically increase the dimensionality of the optimization problem. Making this assumption on the separability of the (marginal) revenue function across the projects represents a trade-off between the generality of the underlying model and my ability to structurally simulate and estimate a model of aggregated investment and employment at the firm level. And while many managerial decisions on strategy and finance in large firms are taken centrally it is common for production, investment and employment decisions to be taken on a more local level in line with decentralised decision making.

### 4.3.3 Aggregation

Given my separable log-linear threshold structure of investment and hiring at the project level I could attempt to model firm level behavior using

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\(^7\)The project and firm level shocks need not be independent but this assumption greatly simplifies the mathematics with little loss of generality. A necessary condition for our aggregation procedure is that these shocks are not perfectly correlated.
probabilistic methods. In particular, the probability distribution of projects between their investment and hiring thresholds at the firm level will obey the Kolmogorov equations used by Caballero (1993), Bertola and Caballero (1994), Eberly (1994) and Bloom (2000). However, while this approach can be attractive for the one dimensional problems modelled in these papers for my two dimensional investment and hiring problem this does not have an explicit analytic solution\(^8\) and would require complex numerical solutions.

So instead I use a simpler but more flexible simulation approach which uses repeated draws of demand shocks at the project level and aggregates up to the firm level. This has the great advantages of enabling me to model aggregation over any number of projects, so that for example I can examine the investment behavior for small firms which operate two production projects only.

\subsection*{4.4 Estimation and Simulation}

In the model above I have a large vector of structural parameters 
\(\{r, \delta_K, \delta_L, \alpha, \gamma, \sigma_F, \mu, \sigma_P, B^*/S^*, E^*/F^*\}\), in which all parameters could potentially be estimated. But to reduce the dimensionality of the estimation procedure, I use values from the extant literature where possible. The firm’s discount rate \(r\) is assumed to be 10\%, capital depreciation \(\delta_K\) is set at 8\%, the labour quit rate \(\delta_L\) is set to 10\%, and the coefficients \(\alpha\) and \((1 - \alpha)\) on capital and labour in production are set at 1/3 and 2/3 respectively\(^9\). Markets

\(^8\)At least not an analytic solution known to the author.
are assumed to be imperfectly competitive with an elasticity of demand\(^{10}\) of -2, so that \(\gamma = 0.5\). The growth rate and variance of firm level sales demand growth, \(\mu\) and \(\sigma_p^2\), are set equal to the sample averages of 6.3\% and 15\% respectively. The remaining three parameters \(\theta = \{\sigma_p^2, B^*/S^*, E^*/F^*\}\) - the degree of plant level uncertainty \(\sigma_p^2\), the ratio of purchase/resale revenue productivities of capital, and \(B^*/S^*\), the ratio of hiring/firing revenue productivities for labour, \(E^*/F^*\) - are then freely estimated\(^{11}\).

The non-linearity of the project level model and the need for simulation and aggregation up to the firm level requires me to use 'indirect inference' estimation procedures to identify structural parameters in the underlying model.

### 4.4.1 Estimation

Conceptually I have a data generating process for investment and hiring, 
\[ I = \{(I_1, H_1)', (I_2, H_2)',...,(I_N, H_N)'\}, \]
which can be written as a function of firm level sales, \(S = \{S_1, S_2,...,S_N\}\), and my vector of free parameters \(\theta\). I label the marginal distribution of \(I\) as \(p(I \mid S, \theta)\). Standard maximum likelihood procedures could be used to estimate \(\theta\) if \(p(I \mid S, \theta)\) was known

\[
\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{N} \ln p(I_i \mid S_i, \theta)
\]

\(^{10}\)This elasticity of demand is taken from assuming a 100\% price markup over costs. This is high in comparison with the evidence on price markups (see, for example, Martins, Scarpetta and Pilat (1996)), but is implied by a price elasticity of 0.5. Lower price markups would require even higher price elasticities with constant price elasticity demand curves, so this represents a trade-off between assumptions over high markups and high price elasticities.

\(^{11}\)The parameter values for purchase and sales revenue productivities only need to be estimated as a ratio rather than as a level.
However, $p(I \mid S, \theta)$ has no known closed form. So I will use an alternative estimation procedure called indirect inference, which has been developed by Gouriérioux, Monfort and Renault (1993) and Smith (1993), and applied by Cooper and Haltiwanger (2000). This procedure involves three steps

1. Define an auxiliary model in which a mis-specified conditional distribution for investment and hiring, $I$, called $g(I \mid S, \Psi)$ is defined, where $\Psi$ has an equal or larger dimensionality$^{12}$ than $\theta$. This function $g(I \mid S, \Psi)$ has a closed analytical form so that using the actual data for $I$ I can estimate $\Psi$ through maximum likelihood:

$$
\hat{\Psi} = \arg \max_\Psi \sum_{i=1}^{N} \ln g(I \mid S, \Psi)
$$

2. Pick a parameter $\theta$ and simulate a set of data for $I$ - where the simulated data is labelled $I(\theta)$ - and estimate $\Psi(\theta)$ using maximum likelihood on $g(I \mid S, \Psi)$. The maximum likelihood estimator is defined as follows

$$
\tilde{\Psi}(\theta) = \arg \max_\Psi \sum_{i=1}^{N} \int \ln g(I \mid S, \Psi)p(I \mid S, \theta)dI
$$

$$
= \arg \max_\Psi \sum_{i=1}^{N} \left( \frac{1}{H} \sum_{j=1}^{H} \ln g(I_j(\theta) \mid S, \Psi) \right)
$$

where the second line is the simulated density estimate using $H$ simulation draws. Lee (1995) proves that this simulated estimator will be fully consistent so long as $H = AN^{1/2}$ where $N$ is the number of observations and $A$ is some fixed constant. I choose $A$ so that $H = 3$.

---

$^{12}$This is the order condition. It is also necessary that the Jacobian of $\Psi$ with respect to $\theta$ has full rank to ensure identification - the rank condition.
where the low number of simulated draws is necessitated by the time intensive nature of each simulation\(^\text{13}\).

3. Repeat step 2 minimising over a grid of parameter values using the criterion \(Q(\hat{\Psi}) = (\hat{\Psi}(\theta) - \Psi)'W^{-1}(\hat{\Psi}(\theta) - \Psi)\). Gallant and Tauchen (1995) suggest using the variance covariance matrix of \(\hat{\Psi}\) from step 1 for \(W\), because this is asymptotically efficient. I can then define my indirect inference estimator of \(\theta\) as

\[
\hat{\theta} = \arg\min_{\theta} (\hat{\Psi}(\theta) - \Psi)'W^{-1}(\hat{\Psi}(\theta) - \Psi)
\]

where

\[
\sqrt{N}(\hat{\theta} - \theta) \rightarrow^d N[0, Q_{\theta}^2(\hat{\theta})VCV(\hat{\Psi})Q_{\theta}^2(\hat{\theta})]
\]

and \(VCV(\hat{\Psi})\) is the variance covariance matrix of \(\hat{\Psi}\) estimated from the actual data.

It is shown by Gallant and Tauchen (1995) that the closer my auxiliary marginal \(g(I | S, \Psi)\) is to the actual marginal \(p(I | S, \theta)\) the more efficient the final estimator. The most efficient moment conditions are the scores of the true density function \(\partial_{\theta}p(I | S, \theta)\). In the limit if these are the same or \(p(I | S, \theta)\) is nested by \(g(I | S, \Psi)\) then this indirect inference is fully efficient.

The two auxiliary regressions I use are: (i) a linear regression of \(I\) on current and lagged values of \(S\), and (ii) a linear regression of \(I\) on a cross sectional spline of the current values of \(S\). The first will capture the important

\(^{13}\)The \(H\) draws for the simulator are only drawn once - that is we use the same seed in each iteration. This ensures that the estimator will converge.
time series dynamics of the investment and hiring processes, while the second will provide a flexible estimator of the cross sectional non-linearities of the investment and hiring response to shocks of different sizes. The results from both estimation procedures, which should be asymptotically equivalent for a correctly specified structural model, are compared and seen to be reassuringly close.

Using a linear auxiliary equation also allows for all maximum likelihood steps to be replaced by standard ordinary least squares (OLS)\textsuperscript{14}. To calculate $Q_{\theta'}(\bar{\theta})$ I take numerical derivatives around the criterion function $Q(\bar{\theta})$ at $\bar{\theta}$.

4.4.2 Simulation

To estimate my parameters we need to construct simulated data for each parameter vector $\theta$. This simulation procedure starts by assuming the investment and hiring behavior of individual projects can be modelled exactly according to the threshold rules laid out in Table 1. Within every firm each project is assumed to experience an idiosyncratic demand shock which is modelled on a weekly basis to allow for continuous investment and hiring throughout the year\textsuperscript{15}. This shock is drawn from a normal distribution with zero mean and variance $\sigma_p^2/52$, yielding an annual variance of $\sigma_p^2$.

\textsuperscript{14}The log-likelihood function is actually a discontinuous function of $\theta$ with this procedure. This does not cause problems from a theoretical point of view since it can be shown that $\ln L(\theta)$ is stochastically equicontinuous. But computationally this requires us to use simulated annealing since this, firstly, is a maximisation over a discrete parameter space, and secondly, can deal with maximisation over functions that are not globally convex.

\textsuperscript{15}The frequency of these demand shocks is not important for the results we present except that in simulations with lower frequency monthly and quarterly shocks we obtain lumpier investment and hiring responses because less temporal aggregation occurs. We choose weekly shocks as a trade-off between their ability to mimic continuous demand uncertainty, which we feel more realistically mimics a firms operating environment, and computational time requirements. Simulations take about four hours on an 800 MHz computer.
Projects are also assumed to be subjected to a firm level shock which is common to all projects in the same firm. This firm level demand shock is proxyd for by the growth in real firm level sales taken from its accounting data. To allow for variation in firm level sales growth within the year I add on a weekly firm level sales shock. This is drawn from a normal distribution with zero mean and variance $\sigma_F^2$/$52$, so that its yearly variance is $\sigma_F^2$. This additional variation in the firm level demand throughout the year also allows me to examine the importance of the assumption made by some papers that aggregated demand grows smoothly within each year\textsuperscript{16}. Reassuringly, this within year variation of the firm level sales growth term has no significant impact upon the simulated investment and hiring responses.

Following Table 1 I assume that workers are hired when the marginal revenue product of labour rises to $E^*$ and are fired when this falls to $F^*$. Workers are also assumed to continuously quit the firm at an exogenous quit rate of $\delta_L = 10\%$ on an annualized basis. Investment only taking place when the marginal revenue product of capital rises to $B^*$ and disinvestment occurs when this falls to $S^*$. Capital is assumed to depreciate at an exogenous rate of 8% per year.

For each firm I also need to start off with an initial distribution of projects between these investment, disinvestment, hiring and firing thresholds - that is an initial distribution across the central region in figure 1. This is generated by starting off all projects as identical and running the simulation for twenty years before using any actual firm level data, with sales growth set equal for each firm to its long run average. This will yield a starting density function for

\textsuperscript{16}See, for example, Caballero (1993) and Bertola and Caballero (1994).
projects equal to the long run ergodic distribution. To remove the influence of this assumption on the initial distribution from the estimation I also drop the first three years of simulated data when examining my simulation results. This three year start up period was chosen as a compromise between longer periods which reduce the impact of this initial assumption on the distribution still further and shorter periods which allow me to use more of my firm level data and minimize any selection bias from taking firms with longer observed duration.

My simulated firm level investment and hiring data is then be calculated by adding up across all projects within each firm. In my simulations I ran three experiments in which my firms are assumed to operate 2, 25 and 250 projects. This wide variation in the degree of aggregation modelled at the firm level allows me to model anything from a small firm operating a couple of production projects up to a large multinational with hundreds of units. Perhaps surprisingly, the degree of aggregation appears to make little difference to my panel results except that greater aggregation smooths the observed investment series and reduces the frequency of zero investment observations.

So in the results presented in Section 5 I use 25 projects per firm.

\(^{17}\)An alternative procedure would have been to analytically calculate the ergodic distribution and draw directly from this, as Caballero (1993), Bertola and Caballero (1994), Eberly (1994) and Bloom (2000) do. However, with two factors (labour and capital) I know of no analytical solution for the partial differential equation describing the ergodic distribution.

\(^{18}\)Three years was also the shortest start up period at which our results were not significantly sensitive to changing this start up period.

\(^{19}\)It appears that panel estimators across firms are already aggregated across so many firms within our Compustat dataset that further aggregation within firms across projects has little additional impact on the panel estimates.
4.5 Data Descriptives

The data is a panel of firm from the US Compustat dataset from 1980-1998 inclusive which was extracted and initially constructed using the same procedure as Bond and Cummins (2000). The firms capital stock was generated using the perpetual inventory method with an 8% depreciation rate and an initial adjustment to control for the effect of inflation on the accounting book value. The data was cleaned to remove mergers and acquisitions by dropping observations with large jumps in the sales, employment and capital stock figures. Accounting periods below ten months and above fourteen months were also dropped. After keeping those firms with four or more consecutive years of data my panel comprised of 3,072 firms with 29,319 observations, and the summary statistics are detailed in the table below.

Table 4.2: Descriptive Statistics for the Compustat Data Set.

<table>
<thead>
<tr>
<th>3,072 firms, with 29,319 observations between 1980 to 1998</th>
<th>mean</th>
<th>median</th>
<th>std. dev.</th>
<th>min.</th>
<th>max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{I_t}{K_{t-1}} ) (investment)</td>
<td>0.157</td>
<td>0.114</td>
<td>0.147</td>
<td>-0.096</td>
<td>1.165</td>
</tr>
<tr>
<td>( \Delta \log y_t ) (real sales growth)</td>
<td>0.063</td>
<td>0.266</td>
<td>0.157</td>
<td>-0.471</td>
<td>1.208</td>
</tr>
<tr>
<td>( \frac{dL_t}{L_{t-1}} ) (employment growth)</td>
<td>0.063</td>
<td>0.047</td>
<td>0.167</td>
<td>-0.562</td>
<td>0.941</td>
</tr>
<tr>
<td>( K_t ) (real capital stock, 1990 $M)</td>
<td>1632</td>
<td>137</td>
<td>5940</td>
<td>0.067</td>
<td>127,032</td>
</tr>
<tr>
<td>( Y_t ) (real sales, 1990 $M)</td>
<td>2037</td>
<td>310</td>
<td>7243</td>
<td>0.434</td>
<td>169469</td>
</tr>
<tr>
<td>( L_t ) (employment, 1000's)</td>
<td>12.8</td>
<td>2.47</td>
<td>38.8</td>
<td>0.05</td>
<td>876.8</td>
</tr>
<tr>
<td>observations per firm</td>
<td>11.4</td>
<td>11</td>
<td>5.32</td>
<td>4</td>
<td>19</td>
</tr>
</tbody>
</table>
4.6 Results

The model was estimated using two auxiliary equations - one based on the time series behaviour of investment and hiring responses to demand shocks, and one based on the cross sectional response of investment and hiring to demand shocks. If my model is correctly specified both approaches will be asymptotically equivalent. In fact, as I show, the results from my Compustat sample are reassuringly similar.

4.6.1 Time Series Estimation

One of the basic stylized facts I would like to replicate is the dynamic response of capital and labour to demand shocks. The most notable features of the actual data are the low response to current demand shocks, and the significant response to lags of past demand shocks. These dynamic responses are examined using the following auxiliary estimating equation

\[
\left( \frac{I}{K} \right)_t \text{ or } \left( \frac{dEmp}{Emp} \right)_t = \alpha + \beta_0(\Delta \log Sales_t) + \beta_1(\Delta \log Sales_{t-1}) + \beta_2(\Delta \log Sales_{t-2}) + \ldots + \beta_{10}(\Delta \log Sales_{t-10})
\]

Following the indirect inference procedure detailed in section (4.4.1) the three free parameters in the model, \( \theta = \{B^*/S^*, E^*/F^*\} \), were estimated by minimizing the weighted distance

\[
\tilde{\theta} = \arg\min_{\theta} (\tilde{\beta}(\theta) - \tilde{\beta})' VCV(\tilde{\beta})^{-1} (\tilde{\beta}(\theta) - \tilde{\beta})
\]

where \( \beta = \{\alpha, \beta_0, \beta_1, \ldots, \beta_{10}\} \) is the vector of OLS parameters, \( \tilde{\beta}(\theta) \) are the simulation based OLS parameters, and \( \tilde{\beta} \) are the actual data OLS parameters. That is, the free parameters \( \theta \), were estimated by minimizing the weighted
distance between the OLS time series parameters in the actual data and the simulated data for my Compustat sample\textsuperscript{20}. The estimated parameter values and standard errors are shown in Table (4.3) below.

Table 4.3: Estimated Parameters.

<table>
<thead>
<tr>
<th>Estimate</th>
<th>E*/F*</th>
<th>Standard Error</th>
<th>B*/S*</th>
<th>E*/F*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>20</td>
<td>0.11</td>
<td>1.1</td>
<td>0.23</td>
</tr>
</tbody>
</table>

To evaluate my simulation models and these estimates Figures (4.2) and (4.3) show the cumulative response of actual and simulated investment and employment to current and past demand shocks.

Actual and simulated investment series both display an impact response to demand growth of around 0.3. This is far below the long run response to demand growth of unity which both series appear to be converging to, suggesting that irreversibilities reduce the impact response of investment to demand growth. In comparison to investment the actual and simulated employment growth responds more rapidly, with a impact point estimate of around 0.6. However, this is also significantly below the long run response coefficient of unity which the series appear to be converging to, suggesting irreversibilities and real options are playing some, albeit a smaller, role here.

The reason in my simulated data that labour displays a larger impact response than capital to demand growth is that the hiring and firing thresholds are much closer together than the investment and disinvestment thresholds. This closeness of the hiring and firing thresholds leads to a stronger imme-

\textsuperscript{20}As the investment and employment series are jointly determined in the model these have to be jointly estimated. To do this equal weight was put on the distance between the time series parameters for the actual and simulated investment and employment series.
Figure 4.2: The Cumulative Response of Investment to Demand Shocks - Actual and Simulated Investment

Note: The graph plots the cumulative estimated response of actual and simulated investment to current and lagged sales changes.
Figure 4.3: The Cumulative Response of Employment to Demand Shocks - Actual and Simulated Employment

Notes: The graph plots the cumulative estimated response of actual and simulated employment changes to current and lagged sales changes.
diate response as a greater distribution of the individual projects will be near to one of their thresholds. This larger immediate response also leads the effects of lagged demand growth on the current distribution of projects between their hiring and firing thresholds to fade much faster. This explains the empirically weaker role of lagged demand growth in explaining current employment growth.

In contrast for capital the investment and disinvestment thresholds are much further apart. As a result within each firm many projects will not be near enough to their investment thresholds to respond immediately to a demand shock, leading to a low impact response. Over time, however, with drift towards the investment threshold due to deprecation these projects will eventually invest, leading to a significant and strong lagged response.

This implies that the response of investment and hiring to micro and macro demand shocks will take time to feed through as all firms, plants and projects slowly adjust their factor demands. This delayed response will vary both by factor and firm according to their recent history of shocks. Firms which have experienced strong demand growth in the recent past will have a large mass of projects near their investment threshold and so respond strongly to current demand growth. Firms which have experienced weak demand growth in the recent past, however, will have few projects near their investment thresholds and so will display a low response to current demand growth.
This would explain the noted instability of investment and labour demand response parameters across firms and across time\(^2\). The approach undertaken by this structural model, which accords well with the time series properties of US firm level data, however, should be robust to this. As such it should help to predict the response to current demand shocks and policy interventions by firm and industry.

### 4.6.2 Cross Sectional Results

Another basic property of the data I would like to replicate is the differential cross sectional response of investment and hiring to sales growth. To demonstrate this Figures (4.4) and (4.5) plot the cross sectional response of investment and labour demand to sales changes in my Compustat data set. This cross sectional response is estimated non-parametrically using a local linear estimator, as suggested by Fan and Gijbels (1996)\(^2\). It can be seen that the investment response displays a notable convexity, with only a limited response to negative sales shocks. The OLS linear estimate of the investment response is also plotted for comparison, from which it can be seen that this can be misleading for large positive or negative sales shocks. The employment response to sales is also convex but to a lesser extent, with a clear firing response to negative sales changes\(^\text{23}\).

\(^2\)For evidence on parameter instability in investment and labour demand equations see, for example, Clark (1979), Chrinko (1993) and Nickell (1984).

\(^2\)Local linear estimators are similar to kernel estimators except they are estimated using a linear term in addition to the kernel local average term. This means they tend to display less bias and have significantly better end point behaviour (see Fan and Gijbels, 1996 or Hurdle, 1992). Nadaraya-Watson kernel estimation provides similar results.

\(^\text{23}\)This convex cross-sectional investment response is also found in UK firm level data (see, Bloom, Bond and Van Reenen, 2001), in US plant level data (see Cooper and Haltiwanger, 2000), and Italian plant level data (see Bond and Lombardi, 2001).
Figure 4.4: The Shape of the Actual Investment Response

Notes: The graph plots the local linear polynomial estimate of the cross sectional response of investment to sales changes. Also plotted is a five knot quadratic spline fit of the local polynomial estimator, and the OLS linear response estimate.
Figure 4.5: The Shape of the Actual Employment Response

Notes: The graph plots the local linear polynomial estimate of the cross-sectional response of employment to sales changes. Also plotted is a five knot quadratic spline fit of the local polynomial estimator, and the OLS linear response estimate.
In Figures (4.4) and (4.5) this cross sectional response is also estimated using a more easily parameterised quadratic spline with five knots. The knots were chosen to ensure the spline estimator would closely match the investment and hiring response found in the data by the more flexible local linear estimator. The close fit between the local-linear and spline estimator suggests that the spline will provide a good auxiliary equation for estimating the cross sectional response of investment and hiring to sales growth.

These simulated investment responses are calculated using the following auxiliary estimating equation

\[
\frac{I}{K}_t \lor \frac{d\text{Emp}}{\text{Emp}}_t = \beta_0 + \beta_1 \Delta \log Sales_t + \beta_2 \Delta \log Sales^2_t + \beta_3 \text{spline}_1 \cdots \beta_5 \text{spline}_5
\]

where \( \text{spline}_j = (\Delta \log Sales_t - K_j)^2 \cdot I(\Delta \log Sales > K_j) \), with \( I(\Delta \log Sales_t > K_j) = 1 \) if \( \Delta \log Sales_t > K_j \) and zero otherwise, and \( K_i \) are the knots.

Again, by following the indirect inference procedure detailed in section (4.4.1), I minimized the weighted distance between the OLS cross sectional parameters in the actual and simulated data, yielding the parameter estimates shown in Table (4.4) below.

<table>
<thead>
<tr>
<th>( \sigma_P^2 )</th>
<th>( \frac{B^<em>}{S^</em>} )</th>
<th>( \frac{E^<em>}{F^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.4</td>
<td>15</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.16</td>
<td>2.6</td>
</tr>
</tbody>
</table>

\( ^{24} \)I can not use the local linear polynomial for my auxiliary equation since it has too many parameters - in fact two parameters for every data point. A linear spline would be a restricted form of the local linear estimator, so that a quadratic spline is similar to a restricted form of the local linear estimator.
These results can also be examined visually in Figures (4.6) and (4.7), where the actual and simulated investment and employment cross-sectional responses are plotted, along with a linear OLS estimate response for comparison. In Figure (4.6) I see that both the actual and simulated investment series display a strongly convex response with a limited response to negative sales terms. However, for very high sales growth terms above about 0.3 (the 98th percentile of sales growth) the actual investment response appears to fall. This suggests that some convex adjustment costs are also present in addition for very large investment episodes. In Figure (4.7) I see that the actual and simulated labour responses look extremely close, suggesting that my simulation model provides a good fit to the actual employment change behaviour of firms.

These response asymmetries occur in the simulated data because positive demand drift, depreciation and labour quitting push the distribution of projects towards their investment and hiring thresholds. This means most firms are more sensitive to positive than negative demand shocks. This asymmetry is much more pronounced for investment because the thresholds are so much further apart. As a result the impact of this demand and depreciation drift towards the investment threshold will play a powerful role in reducing the response to negative demand shocks by leaving very few projects near their dis-investment thresholds.

The response convexity in the investment response occurs because of the mix of inter and intra marginal response by projects. For small shocks only a few projects will respond leading to a limited investment response. As the size of the shock grows, however, not only will those projects invest more,
Figure 4.6: The Shape of the Actual and Simulated Investment Response

Note: The graph plots the spline estimate of the actual and the simulated cross sectional response of investment to sales changes. Also plotted for comparison is the OLS linear response estimate.
Figure 4.7: The Shape of the Actual and Simulated Employment Response

Note: The graph plots the spline estimate of the actual and the simulated cross-sectional response of employment to sales changes. Also plotted for comparison is the OLS linear response estimate.
but more projects will also start to invest. This mix of more investment by projects already investing and more projects investing, leads to an increasing and convex response to demand shocks. Since labour has a smaller gap between the hiring and firing thresholds these aggregation effects play a less important role, since more projects are near their employment thresholds. As a result the convexity of the employment response to sales growth is less strong.

From Figures (4.6) and (4.7) it is also clear that using a linear OLS estimator of the response to demand growth would over estimate the disinvestment and firing response to bad shocks and under estimate the investment and hiring response to good shocks. This could lead to serious prediction errors for episodes with large demand changes. Given the non-linearity of the investment response these under or over predictions would be most likely to occur during booms or busts when demand is rapidly changing.

4.6.3 Extensions

Two extensions of this approach will be examined in future work:

1. First, I can use firm level information on uncertainty to vary their investment and employment thresholds in the cross section. It has been shown before by Guiso and Parigi (1999) and Bloom, Bond and Van Reenen (2001) that higher degrees of firm level uncertainty reduce firm's response to demand shocks as real option values increase. A similar

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25 Interestingly, a similar conclusion, that aggregation over non-convex micro level projects will outperform at turning points of the economic cycle, has also been reached by Caballero, Engel and Haltiwanger (1995) and Cooper, Haltiwanger and Power (1999) using a different methodology.
effect of uncertainty on reducing firm’s investment and employment response, but acting through the effects of delivery lags, has also been made by Nickell (1977). These propositions can be structurally tested and evaluated in this model by matching in share base returns uncertainty measures to predict firm level investment and hiring thresholds.

2. Second, I can use this approach to undertake some explicit policy experiments and evaluations. For example, data on the cost of capital can be used to examine to what extent this model can explain aggregate variations in investment and employment growth. Alternatively, this approach could be used to examine the effects of changing labour hiring and firing costs on firms’ investment and employment decisions.

4.7 Conclusions

This paper studies the effects of real options and irreversibilities on firm’s joint investment and employment decisions. A simulation model is constructed and its structural parameters estimated by indirect inference on US firm level data. The predicted time series and cross sectional responses of the model are shown to closely mimic those of firms in the US data set. In particular the strong dynamics in the investment response to lagged demand shocks and the convexity of its cross sectional response. Given the structural nature of this estimator it should provide a useful tool for modelling the effects of policy interventions on firms’ investment and employment decisions under different degrees of uncertainty.
Chapter 5

Real Options, Patents, Productivity and Market Value: Evidence from a Panel of British firms

5.1 Abstract

Patents citations are a potentially powerful indicator of technological innovation. Analysing the new IFS-Leverhulme database on over 200 major British firms since 1968 we show that patents have an economically and statistically significant impact on firm-level productivity and market value. We also find that while patenting feeds into market values immediately it appears to have a slower effect on productivity. This is potentially because of the need for costly investment in new equipment, training and marketing required to embody patents into new products and processes. This may generate valuable real options because patents provide exclusive rights to develop new innovations, thereby enabling firms to delay their investments. We find that higher market uncertainty, which increases the value of real options, reduces the
impact of new patents on productivity. These real options effects have implications for the role of macro and micro stability in the take up of new technologies and productivity growth.
5.2 Introduction

There is a consensus that technological advance is crucial in the "new economy". But measuring technology has always been one of the most perplexing problems facing empirical economics. One tradition, epitomised by Solow (1957), is to measure technology as a residual from a production function. The problem is that the residual, no matter how cleverly constructed, is rather like a statistical dust bin - holding a lot of trash as well as a few nuggets of gold. A second tradition, which this paper follows, is to construct observable proxies for technical change. The most popular measure of technology is research and development expenditures (R&D). Unfortunately at the firm level there was no requirement to report R&D expenditures in Britain before 1989, so this hampers the generation of a long time series. Innovation counts have been frequently used in the UK, but the best series for these ended in 1983 (see Pavitt, Robson and Townsend, 1987, Blundell, Griffith and Van Reenen, 1999 and Geroski, 1990).

Counts of patents have also been a popular choice to proxy innovation. And patents themselves contain a wealth of other information (e.g. Lerner, 2000). In particular, the front of a patent details other patents which contributed to the knowledge underlying the new patent. This information can be used in a variety of different ways. We start off with the most obvious use. A patent which is cited many times is more likely to be valuable than a patent which is rarely cited (Griliches, 1990). Other researchers have used patent citations as a "paper flow" to track the way knowledge spills over between organisations and areas (Jaffe, Trajtenberg and Henderson, 1993;
Jaffe and Trajtenberg, 1998) and this is a route that we have pursued in complementary work.

We look at the impact of patents on two measures of company performance - productivity and market value. Production functions are more easily interpretable and comparable with other work. Market value is a more forward looking measure, which has attractions for the analysis of an activity whose pay-off may not be for many years in to the future. There is a small literature emerging on the impact of patent citations on company performance, but all the existing work that we know of is based on U.S. firms (e.g. Hall et al, 2000)\(^1\).

From our preliminary work with the data it became apparent that while patents have an immediate impact upon market values they take time to affect productivity. One potential explanation is that the new products and processes which are covered by the patents have to be embodied in new capital equipment and training. Firms may also need to undertake expensive marketing and advertising to promote their new products. As such, this will involve extensive sunk cost investments - these capital, training and marketing outlays will be (at least partially) irreversible. But since patents provide firms with the exclusive rights to their new technologies they have the option to wait until making these sunk costs investments. When market conditions are uncertain this will generate valuable real options. Therefore, by giving firms a legally protected right to delay investing, patents provide an excellent test of the importance of real options.

\(^1\)There are, of course, several econometric studies of the impact of patent counts on British firm performance (e.g. Bosworth, Greenhalgh and Stoneman, (2000) and Bosworth, Greenhalgh and Longland (2001)).
We adapt the developing real options literature to explain the take up of new products and processes covered by patents. The theories developed in this paper predict that higher market uncertainty will lead firms to be more cautious about their investments. We use this theory to then derive empirical predictions on the relationship between patents and uncertainty and empirically test them.

The structure of this paper is as follows. Section 2 describes the database that we have constructed and some of its key features. Section 3 sketches some simple models and the real options extensions that we use to estimate the effects of patenting on company performance. Section 4 details the econometric results and section 5 gives some concluding comments. In short, we find considerable evidence of the importance of technology for firms' productivity and stock market performance. Higher uncertainty, as predicted, reduces this effect of patents on productivity but appears to have no significant effect on market value.

5.3 Data

We combine three principle datasets in constructing the IFS-Leverhulme database. Full details of the matching between the datasets is contained in the Appendix, but we sketch the process here. The first dataset is the Case Western Patent data, the second is the Datastream annual company accounting data, and the third is the Datastream daily share returns data.

To construct the patents data base we used the computerised records

\footnote{See, in particular, Dixit and Pindyck (1994), Eberly and Van Mieghern (1997), and Bloom (2000).}
of patents granted in the U.S. between 1968 and 1996. This is the largest electronic patent dataset in the world (the European Patent Office records begin only in 1976, and the records are patchy until the mid 1980s). The data is held in Case Western and we received considerable help from their staff in setting up the files. Practically all major patents are taken out in the U.S., so we are screening out many low value patents by following this strategy.

The second and third datasets contain the accounts and share returns of firms listed on the London Stock Exchange. From the population of public firms we selected those whose names began with the letters ‘A’ through ‘L’, which represents a random sample from the whole population. We also added in the top 100 R&D performing firms in the UK that were not already included in this list to maximise the numbers of patents we could collect. Ideally we would have collected information on all firms on the Stock Market, but the resource cost was too great. For all of these 415 firms we used ‘Who Owns Whom’ from 1985 to find the names of all subsidiaries. We then used these subsidiary names to match to the Case Western Dataset by name.

5.3.1 Patents and Citation Data

The intersection of the two datasets gave us 236 firms who had taken out at least one patent between 1968 and 1996. The total number of patents taken out by this group over the entire period was 59,919, representing about 1% of the 6 million patents ever taken out at the U.S. Patent Office. Table

\[\text{There are many problems with only using one year of data to match in the corporate structure. Clearly this changes over time, albeit slowly for most firms. The process of matching is, however, extremely labour-intensive so it was only practical to perform it for one year. In future work we intend to also do the matching for later and earlier years.}\]
(5.1) below shows that most of our group of UK firms are involved in only a modest amount of patenting with about half the sample receiving more than 25 patents, while 12 firms received over 1000 patents during the period. This concentration of innovative activity within large firms (the 12 account for 72% of all patents in our data), reflects a similar phenomena in R&D expenditure where the 12 largest enterprises account for about 80% of all R&D expenditure.

Table 5.1: The Distribution of Firms by Total Patents 1968-1996.

<table>
<thead>
<tr>
<th></th>
<th>1 or more</th>
<th>10 or more</th>
<th>25 or more</th>
<th>100 or more</th>
<th>250 or more</th>
<th>1000 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms</td>
<td>236</td>
<td>161</td>
<td>117</td>
<td>75</td>
<td>41</td>
<td>12</td>
</tr>
</tbody>
</table>

Table (5.2) reports the patenting activity of the twelve largest patenters. This selection of firms reflects the strong performance of the chemicals, pharmaceuticals and the defence engineering sectors in the UK.

Table 5.2: The Top 12 Patenting Firms.

<table>
<thead>
<tr>
<th>Firm</th>
<th>No. Patents</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICI</td>
<td>8422</td>
</tr>
<tr>
<td>Shell</td>
<td>7200</td>
</tr>
<tr>
<td>Smithkline Beecham</td>
<td>3672</td>
</tr>
<tr>
<td>BP</td>
<td>3632</td>
</tr>
<tr>
<td>BTR</td>
<td>3432</td>
</tr>
<tr>
<td>Lucas Industries</td>
<td>3119</td>
</tr>
<tr>
<td>GEC</td>
<td>3054</td>
</tr>
<tr>
<td>Hanson</td>
<td>2892</td>
</tr>
<tr>
<td>Unilever</td>
<td>2644</td>
</tr>
<tr>
<td>Siebe</td>
<td>1876</td>
</tr>
<tr>
<td>Rolls-Royce</td>
<td>1575</td>
</tr>
<tr>
<td>Glaxo Wellcome</td>
<td>1528</td>
</tr>
</tbody>
</table>
The patents are graphed by their year of application in Figure (5.1). The lesser degree of patenting activity in the latter part of the period reflects truncation bias (on the right) because we collect statistics on patents granted. Since there is a delay between applying for and granting a patent of about two years, this leads to a downward bias towards the end of the period. There is also a truncation on the left of the graph as there may have been patents granted post 1968 which were applied for pre-1968. These caveats apart, there is little discernible trend in the total patents numbers granted to UK firms. There was some decline 1968-1983, a pick-up 1983-1990 and then decline in the 1990s. Interestingly, this broadly mirrors the growth rates in UK productivity.

We also have data on the citations made by any of the other 6 million patents in the main data set to our sample of 59,919 patents. Citations can be taken as an indicator of the technological value of a patent in that those patents which are frequently cited are likely to be more innovative and technologically productive. In Figure (5.2) we plot the histogram of the lag between a patent being taken out and the subsequent citations to that patent.

It is clear that citations tend to happen relatively early on in a patent’s life when the patent is widely known but technologically still innovative. Interestingly this citation lag still has not completely tailed off even after 20 years. Figure (5.3) plots the histogram of the number of cites per patent from which it is clear that many patents are never cited at all, with a small tail of patents which are frequently cited.

The five most cited patents are tabulated in Table (5.3) below with their
Figure 5.1: Patents per Year

frequency of patents by year

no. patents from that year

application year

application year of the patent


0 1000 2000 3000
Figure 5.2: The Lags Between Patenting and Citing
Figure 5.3: Distribution of Cites Per Patent

Histogram of number of cites per patent
patenting topic, the year they were granted and the number of cites made to them over the period 1976 until 1996.

Table 5.3: The Top Five Cited Patents.

<table>
<thead>
<tr>
<th>Company</th>
<th>Patent Topic</th>
<th>Grant Year</th>
<th>Cites 1976-96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell</td>
<td>Synthetic Resins</td>
<td>1972</td>
<td>221</td>
</tr>
<tr>
<td>Grand Metropolitan</td>
<td>Microwave heating package</td>
<td>1980</td>
<td>174</td>
</tr>
<tr>
<td>ICI</td>
<td>Herbicide compositions</td>
<td>1977</td>
<td>130</td>
</tr>
<tr>
<td>Unilever</td>
<td>Anticalculus composition</td>
<td>1977</td>
<td>97</td>
</tr>
<tr>
<td>British Oxygen Corp.</td>
<td>Pharmaceutical Drugs</td>
<td>1975</td>
<td>89</td>
</tr>
</tbody>
</table>

The total number of citations to our patents, dated by the application year of the patent being cited, is plotted in Figure (??). Because data on citations is only collected for patents granted after 1976 there is an early downward bias reflecting the fact that for patents granted pre 1976 some of the initial citations data is missing. The discussion in the paragraph above and Figure (5.2) suggests that the loss of these early citations could lead to a serious downward bias for patents taken out pre 1976 since for them this would represent a period of relatively high citation activity. There is also a tail end bias as patents applied for towards the end of the period will only be part of the way through their citations lifecycle, and so will have been cited less often by 1996.

To deal with these biases we use a non-parametric series estimator based on a full Fourier sine and cosine expansion. Following the approach in Hall et al (2000) we assume that the total lifetime number of citations per year is constant through out our sample. Therefore any observed changed in the observed aggregate citation levels is due to time varying levels of truncation bias. We assume that this time varying truncation bias varies smoothly over
time according to some piecewise continuous function of time\(^4\). Our nor­
malization estimator then uses a Fourier expansion to fit a smooth curve to
the observed time variation in aggregate citation levels to non-parametrically
estimate a truncation bias function.

A Fourier expansion was used because of its ability to approximate to an
arbitrary degree of accuracy any piecewise-continuous function (see Churchill
and Brown, 1987). We used the first four sine and four cosine terms for
an expansion with the base periodicity set at the total time observation
period of 30 years\(^5\). The smoothing property of our estimator can be seen in
Figure (??) which plots the actual citation frequency and our non-parametric
functional estimator. This functional estimator of the time varying citation
bias is then inverted to re-weight the citations per patent. This ensures that
the normalized citations per patents remain approximately constant over the
period.

In calculating a patent based proxy for knowledge stocks it is also more
sensible to use a stock measure rather than a flow measure of knowledge as the
benefits from a patent are likely to persist into future years. We calculate
a set of preferred measures of the stock of patents through the perpetual
inventory method so that

\[
(Patent \ Stock)_t = (1 - \delta) \times (Patent \ Stock)_{t-1} + Patents_t \tag{5.1}
\]

\(^4\)That our observed citation frequency is not smooth over time, even in our sample of
almost 60,000 patents, is testament to the extreme skew of the citations data. In data
sets such as these which have large second moments the usual weak convergence of the
empirical distributions to their underlying distribution is extremely slow (see for example
Billingsley, 1986), so that smoothing is usually required.

\(^5\)Increasing the length of the base period or using the first three or five terms does
not have any significant impact on our results. This is because the first few terms of the
Fourier expansion drive the results, as noted for example, by Bertola and Caballero (1994)
in a related application.
Figure 5.4: Citations Per Year

Actual and Normalizing Mean Total Cites Per Patent

application year
where the knowledge depreciation rate, $\delta$, is set to 30% as in, for example, Griliches (1990). The first year is calculated by assuming a prior steady state growth of patents of 5%. The same perpetual inventory method is used to calculate the citation stock where the flow variable is the citation weighted number of patents. The “5 year cite stock” uses only the first 5 years of citations (after an application) to obtain a citation weighting but without any normalization. Since we select our citation estimation period to run up to 1990 only whilst our citing data runs up to 1996 this means we have 5 years of observations on citations for every patent so that no truncation bias correction will be needed for this 5 year measure. This reduction in the number of years for estimation thus allows us to compare our normalized full citation weighted patent measure and the five year citation measure.

It is comforting that our three measure of the knowledge stock - the patent stock, the citation weighted patent stock, and the 5 year citation weighted stock - have a strong correlation as demonstrated in Table (5.4) below. This suggests that whilst each should have its own merit in capturing various aspects of the knowledge stock they proxy a similar measure of the technological innovation stock.

<table>
<thead>
<tr>
<th></th>
<th>Pat Stock</th>
<th>Cite Stock</th>
<th>5 Year Cite Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patent Stock</td>
<td>1</td>
<td>0.9871</td>
<td>0.9665</td>
</tr>
<tr>
<td>Cite Stock</td>
<td>1</td>
<td>1</td>
<td>0.9714</td>
</tr>
<tr>
<td>5 Year Cite Stock</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.4: Correlations Between Knowledge Stock Measures.
5.3.2 Firm Level Accounting and Uncertainty Data

The company data is drawn from the Datastream on-line service and represents the accounts of firms listed on the UK stock market. Our initial sample of 415 firms (those whose names began with A-L or were large R&D performers)⁶ for which we matched patent data was then cleaned for estimation. Cleaning involved ensuring that there are no missing values on sales, capital or employment, deleting firms with less than three consecutive observations, breaking the series for firms whose accounting period fell outside 300 to 400 days due to changes in year end timing, and excluding observations for firms where there are jumps of greater than 150% in any of the key variables (capital, labour, sales). After cleaning we were left with a sample of 404 firms, to which 211 were matched as having patenting subsidiaries (see the Data Appendix for details of this matching process)⁷.

Table (5.5) reports summary statistics for this set of 185 patenting firms. Because these are quoted firms the median size is large with sales of about £360 million (in 1985 prices) and a work force of over eight thousand employees. From the last row of the table it can also be seen that we generally have a long time series of data on each firm - on average over 20 years for each firm. However, because of the need to deal with the biases discussed above in patent and citation counts we only use the data up to 1993 for raw patent numbers and 1990 for citations. Hence, the average number of observations per firm is 19 and 16 for patents and citation weighted patents, which

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⁶See the data appendix for more details on the selection of this sample.
⁷This is less than our group of 228 patenters because of both the loss of some firms due to trimming and because of the loss of some years of observations for the remaining firms due to the unavailability or poor quality of data on employment in the early 1970s.
still represents a relatively long time period per firm. The patent numbers demonstrate the large variation in patenting per firm year with some firms only taking out sporadic patents - as demonstrated by the zero patent observations - and others taking out 409 patents in a single year (ICI in 1974). The total cites number represents the normalized sum of citations for all patents taken out in each firm year.

Table 5.5: Descriptive Stats. for the 185 Patenting Firms, 1969-1996.

<table>
<thead>
<tr>
<th></th>
<th>median</th>
<th>mean</th>
<th>stan. dev.</th>
<th>min.</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>real capital (£m)</td>
<td>143</td>
<td>744</td>
<td>1,777</td>
<td>1.6</td>
<td>18,514</td>
</tr>
<tr>
<td>employment (1000s)</td>
<td>8,398</td>
<td>24,374</td>
<td>42,078</td>
<td>40</td>
<td>312,000</td>
</tr>
<tr>
<td>real sales (£m)</td>
<td>362</td>
<td>1,224</td>
<td>2,494</td>
<td>1.15</td>
<td>20,980</td>
</tr>
<tr>
<td>real market value (£m)</td>
<td>153</td>
<td>740</td>
<td>1,766</td>
<td>0.29</td>
<td>19,468</td>
</tr>
<tr>
<td>patents</td>
<td>3</td>
<td>12.6</td>
<td>34</td>
<td>0</td>
<td>409</td>
</tr>
<tr>
<td>total cites</td>
<td>13.7</td>
<td>61.2</td>
<td>157</td>
<td>0</td>
<td>1808</td>
</tr>
<tr>
<td>patent stock</td>
<td>10</td>
<td>42.6</td>
<td>113</td>
<td>0</td>
<td>1218</td>
</tr>
<tr>
<td>cite stock</td>
<td>49.2</td>
<td>202</td>
<td>507</td>
<td>0</td>
<td>5157</td>
</tr>
<tr>
<td>5 year cite stock</td>
<td>26.2</td>
<td>105.9</td>
<td>227</td>
<td>0</td>
<td>2919</td>
</tr>
<tr>
<td>uncertainty</td>
<td>1.39</td>
<td>1.47</td>
<td>0.42</td>
<td>0.60</td>
<td>6.6</td>
</tr>
<tr>
<td>observations per firm</td>
<td>22</td>
<td>20</td>
<td>7.6</td>
<td>3</td>
<td>29</td>
</tr>
</tbody>
</table>

Notes: Capital, sales and market value are all in 1985 £1,000,000s. ‘Patents’ is the total number of patents per firm year whilst cites is the normalized total number of cites to a firms patents per year. Uncertainty is the % standard deviation of daily share returns. Sample covers years 1968-96.

In measuring uncertainty we have to capture measure firms uncertainty about future prices, wages rates, exchange rates, technologies, consumer tastes and government policies. In an attempt to capture all factors in one scalar proxy for firm level uncertainty we use the variance of the firm’s daily stock returns\(^8\), denoted \(\sigma^2_t\). In accordance with theories of real options this

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\(^8\)This measure of uncertainty is also used by other papers in the literature on uncertainty and investment, such as Leahy and Whited (1998).
is a time invariant but firm specific proxy for uncertainty\textsuperscript{9}. This measure includes on a daily returns basis the capital gain on the stock, dividend payments, rights issues, and stock dilutions. Such a returns measure provides a forward looking proxy for the volatility of the firm's environment which is implicitly weighted in accordance with the impact of these variables on profits. A stock returns-based measure of uncertainty is also advantageous because the data is accurately reported at a sufficiently high frequency to provide an extremely accurate measure. Our sampling size of 265 recordings per year for the 22 year life of our average firm therefore provides an extremely low sampling variance\textsuperscript{10}.

5.4 Models of Patents and Company Performance

We work with a simple Cobb-Douglas production function of the form

\[ Q = AG^\alpha N^\beta K^\gamma \]  \hspace{1cm} (5.2)

where \( Q \) is real sales, \( G \) is the knowledge stock, \( N \) is number of employees and \( K \) is the capital stock and \( A \) is an efficiency parameter. Taking logs and introducing subscripts for firm \( i \) at time \( t \) we have

\[ \log Q_{it} = \log A_{it} + \alpha \log G_{it} + \beta \log N_{it} + \gamma \log K_{it} \]  \hspace{1cm} (5.3)

\textsuperscript{9}The real options literature, surveyed in Dixit and Pindyck (1994) for example, always assumes a time invariant uncertainty measure. This drives us to choose a time invariant uncertainty measure so that we can make a close link with the theoretical literature.

\textsuperscript{10}For example, Andersen and Bollerslev (1998) use high frequency exchange rate data with 288 recordings per period and calculate the implied measurement errors are less than 2.5\% of the true volatility.
We parameterise efficiency, \( A_{it} = \exp(\eta_i + \tau_t + \nu_{it}) \), as a function of firm specific fixed effects \((\eta_i)\), time effects \((\tau_t)\) and a random stochastic term \((\nu_{it})\). In our empirical application we use patent stocks and citation-weighted patent stocks (PAT) as empirical proxies of \( G \), the knowledge stock.

\[
\log Q_{it} = \alpha \log PAT_{it} + \beta \log N_{it} + \gamma \log K_{it} + \eta_i + \tau_t + \nu_{it} \tag{5.4}
\]

We estimate equation (5.4) by within groups (least squares dummy variables) correcting the standard errors for heteroscedasticity.

Market value equations are less well established than production functions. The standard approach pioneered by Griliches (1981) is based on a specification of the form (see also Hall et al., 2000 and Bosworth, Greenhalgh and Wharton, 2000)

\[
\log \left( \frac{V}{K} \right)_{it} = \delta \left( \frac{G}{K} \right)_{it} + \tilde{\eta}_i + \tilde{\tau}_t + \tilde{\nu}_{it} \tag{5.5}
\]

where \( V \) is the market value of the firm. The left hand side of equation (5.5) is essentially Tobin’s average \( Q \). Under perfect competition one would expect this ratio to be equal to unity. The deviation of Tobin’s \( Q \) from unity is, in this framework, driven by the fact that the firm possesses intangible \((G)\) as well as tangible \((K)\) capital. “New economy” firms with high levels of intangible knowledge capital will have a much higher market value than one would expect if we merely used their fixed capital stock.

### 5.4.1 Uncertainty and Real Options

The two models laid out above assume that the knowledge contained in patents can be immediately be used and acted on by firms. Patents, however,
represent new products or process innovations whose introduction can involve sizeable investments in additional plant and equipment, hiring and retraining workers, and advertising and marketing. Much of this expenditure will be irreversible - once it is undertaken the initial costs will not be recoverable. Thus, when firms are facing uncertain market conditions then they will posses patent real options. These patent real options reflect the value a firm places on its ability to choose the timing of its investment in its patented technologies when this involves sunk costs.

A large theoretical literature has grown up from the seminal papers of Bertola (1988), Pindyck (1988), and Dixit (1989) demonstrating the important role such real options can play in firm’s optimal investment strategies, with real options even able to account for more than half a firm’s value in uncertain market conditions. As a result real options should play an important role in our approach to modelling investment in innovation. This work emphasizes the role real options play in retarding the response of firms to changing market conditions. When market conditions are uncertain firms become reluctant to commit large sums to new investment projects or dismantle old investment projects in case conditions change. This leads to a ‘cautionary’ investment behaviour. This ‘cautionary’ effect of real options in retarding the response to changing market conditions has been confirmed empirically for physical investment by Guiso and Parigi (1999) in a cross

\[\text{\cite{Bakshi and Larsen 2001}}\]

\[\text{\cite{Abel and Eberly 1996, Dixit and Pindyck 1996, and Bloom 2000}}\]
section of Italian firms, and by Bloom, Bond and Van Reenen (2001) in a panel of UK firms.

The incorporation of new products and processes into a firm’s production schedule will also be subject to precisely this kind of cautionary effect because of the capital investment, training and marketing required. Hence, firms may be reluctant to exploit these patents in uncertain market conditions when the chances of making an expensive mistakes is high. Since patents provide firms with the exclusive right to use their new innovations they have considerable ability to delay their investments generating potentially substantial real options values.

To incorporate these real options effects we extend the concept of our knowledge stock $G$ into embedded knowledge, $G_E$ and dis-embedded knowledge $G_D$, where $G = G_E + G_D$. Embedded knowledge represents those product and process innovations which the firm has invested in. Dis-embedded knowledge, however, represents the remaining ideas which the firm has under patent but has not yet committed into actual production.

When conditions are highly uncertain the firm will be more cautious because of the value of the real options associated with embedding new innovations into production. To model this empirically we define $\lambda(\sigma) = \frac{\partial G_E}{\partial G} \cdot \frac{G}{G_E}$ to be the elasticity of embedded knowledge to total knowledge, where this ratio depends on the uncertainty of a firm's business conditions, $\sigma$. This elasticity of embedded to total knowledge, $\lambda(\sigma)$, will be a decreasing function of uncertainty. More uncertain conditions will increase the cautionary effect of real options and lead to a slower pass through of new innovations into the
Adapting our earlier production function to incorporate these real options effects and using lower case to denote logs we can therefore write

\[ q = a + ag_E + \beta n + \gamma k \]  

(5.6)

where the embedded knowledge stock, \( g_E \), rather than the total knowledge stock, \( g \), is included in the firm's production function. To evaluate the additional effects of uncertainty on production we take a second order Taylor series expansion of the logged production function in the total knowledge stock, \( g \), and uncertainty \( \sigma \). This can be used to predict the qualitative effects that uncertainty should play in the adoption of new technologies.

Firstly, holding other factors constant greater levels of total knowledge will lead to a greater level of productivity as some proportion of new innovations will always become embedded in new products and processes:\footnote{This is true both in the short run and the long run because innovation in general, and patents specifically, generally have a falling private value over time. Hence, slow incorporation reduces their overall value both in the short run and in the long run.}

\[
\frac{\partial q}{\partial g} = \alpha \frac{\partial g_E}{\partial g} = \alpha \lambda(\sigma) \]

(5.7)

\[ > 0 \]

Secondly, this effect of new innovations on output will be falling in the level of uncertainty because this reduces the rate of incorporation of new ideas into production

\[
\frac{\partial^2 q}{\partial g \partial \sigma} = \alpha \frac{\partial \lambda(\sigma)}{\partial \sigma} \]

(5.8)

\[ < 0 \]

\footnote{We assume new innovations have an embodied value drawn from some distribution in which the most valuable innovation will always be embodied.}
Finally, the direct effect of uncertainty is ambiguous. While uncertainty reduces the speed with which new firms embed new innovations into production, there is an ambiguous relationship between the absolute level of embedded innovations and uncertainty

\[ \frac{\partial q}{\partial \sigma} = \alpha \frac{\partial g\theta}{\sigma} \]

\[ \leq 0 \]  

To empirically investigate these effects we include a direct uncertainty term and an uncertainty patenting interaction term. The uncertainty patenting interaction term will then pick up the negative cautionary effect of the term in equation (5.8).

Our estimating equation for the production function will take the form

\[ \log Q_{it} = \alpha \log PAT_{it} + \beta \log N_{it} + \gamma \log K_{it} + \psi \sigma_i + \chi[\sigma_i * \log PAT_{it}] + \eta_i + \tau_i + \nu_{it} \]  

where the coefficients \( \psi \) and \( \chi \) will pick up the direct and interaction effects of uncertainty. Note that we will not be able to separately identify the linear effect of uncertainty from \( \eta_i \) in the specifications where the latter are treated as fixed effects.

A slower uptake of technologies due to real options should have a much less dramatic effect on the forward looking market value measures. If the adoption of new patented technologies is only delayed by real options effects, then as a forward looking measure, market values should only display a limited response reflecting the limited changes to the total expected discounted cash flow. Therefore, in our empirical equation market value equation shown

---

16While it is likely this sign is negative because greater uncertainty reduces the rate of adoption of technology it may not reduce the absolute level of adopted technology.
below, which includes uncertainty interactions, we would expect a lesser and possibly

\[ \log \left( \frac{V}{K} \right)_{it} = \delta \left( \frac{G}{K} \right)_{it} + \theta \sigma_i + \zeta \left[ \sigma_i \ast \log \left( \frac{G}{K} \right)_{it} \right] + \tilde{\eta}_i + \tilde{\tau}_i + \tilde{\nu}_{it} \]

insignificant point estimate on the interaction term \( \zeta \), while the sign of \( \theta \) will as before remain ambiguous.

5.5 Results

Table (5.6) presents the results of estimating a standard production function on our sample of firms. Column (1) has the OLS estimates of the production function for our complete population of over 2,000 Datastream firms. As expected the coefficients on capital and labour are both positive and significant at conventional levels, and their sum is close to unity (suggesting constant returns in tangible factors). In column (2) we undertake estimation with our preferred within groups estimator which controls for time invariant difference between firms by including firm dummies. Again the coefficients on capital and labour are positive and significant, although slightly smaller than in column (1). Columns (3) compares these within groups results from the whole Datastream sample to our sub-sample of patenters. The higher point estimates on capital and lower point estimates on labour imply that our sample of patenting firms are on average more capital intensive than lower tech firms (as one would expect). In fact our patenting firms have on average a 20\% higher capital to labour ratio than non patenting firms.
Table 5.6: Basic Production Functions.

<table>
<thead>
<tr>
<th>Log Real Output</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms</td>
<td>All</td>
<td>Patenters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Capital</td>
<td>0.330***</td>
<td>0.289***</td>
<td>0.437***</td>
<td>0.439***</td>
<td>0.468***</td>
<td>0.471***</td>
<td>0.468***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Log Employment</td>
<td>0.650***</td>
<td>0.606***</td>
<td>0.558***</td>
<td>0.554***</td>
<td>0.502***</td>
<td>0.491***</td>
<td>0.502***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.031)</td>
<td>(0.032)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Log Patent Stock</td>
<td></td>
<td>0.024***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.018)</td>
</tr>
<tr>
<td>Log Cite Stock</td>
<td></td>
<td>0.030***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.039***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>Log 5 Year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.031***</td>
<td></td>
</tr>
<tr>
<td>Cite Stock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Firm Dummies</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Adj. R-Squared</td>
<td>0.901</td>
<td>0.989</td>
<td>0.992</td>
<td>0.992</td>
<td>0.992</td>
<td>0.992</td>
<td>0.992</td>
</tr>
<tr>
<td>No. Observations</td>
<td>18,068</td>
<td>18,068</td>
<td>2219</td>
<td>2219</td>
<td>1896</td>
<td>1896</td>
<td>1896</td>
</tr>
<tr>
<td>No. Firms</td>
<td>2063</td>
<td>2063</td>
<td>211</td>
<td>211</td>
<td>189</td>
<td>189</td>
<td>189</td>
</tr>
</tbody>
</table>

NOTES: The dependent variable is 'log real output'. Columns (1) and (2) present results using our complete Datastream population of all firms. Columns (3) to (7) present the results for our sub-sample of firms with patents. The estimation period covers 1968 until 1993 inclusive for columns (1) to (4), and 1968 until 1990 inclusive for columns (4) to (7) which use the citation data. The *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors are robust to heteroskedasticity.

The last four columns of Table (5.6) reports the results from including patents as a proxy for knowledge in the production function. In column (4) we use patent stocks, in column (5) citation weighted patent stocks and in column (6) the “five year ahead” citation weighted patent stock measure. On all the alternative measures, patent stocks are significant at the 0.05 level with an elasticity of about 0.03. This suggests that a doubling of the patents stock would lead to a 3% increase in total factor productivity. In column (7)
we include both the patent stock and the citation knowledge stock and find that patents are no longer significant. Thus, citations provide significant information over and above raw patents numbers. This suggests citations could provide a valuable proxy for evaluating knowledge stocks and tracing knowledge flows.

Table (5.7) reports the results of estimating the impact of patents on firm market value using the conventional average Q specification described in equation (5.5). In column (1) we use the patent stock measure, in column (2) our citation weighted patents stock measure, and in column (3) the "five year ahead" measure and find all three have significant explanatory power at the 5% level. The coefficient in column (2) suggests, for example, that doubling the citation weighted patents stock would increase the value of firms per unit of capital by about 43%. This large estimate of the effect of cited patents on market values captures the market's expectation of the total discounted rents from patented innovations. These results are larger than those reported for US firms by Hall et al. (2000) where they report coefficients of 0.607 and 0.108 on (patent/capital stock) and (cite patent/capital stock). This appears to be mainly because they chose a 15% rather than a 30% depreciation rate on patents so that their patenting and citation stocks will be approximately twice our size\textsuperscript{17}. Finally, in column (4) we again compare the predictive power of patents and citation weighted patents and find that citations provide

\textsuperscript{17}All our results are robust to using this alternative assumption on the knowledge depreciation rate. For example, if we use a 15% rather than a 30% depreciation rate and re-estimate our market value equations we obtain a coefficient (standard error) of 0.879 (0.327) and 0.246 (0.081) on the (patent stock/capital stock) and (citation stock/capital stock) terms respectively. In our productivity equations we obtain a coefficient (standard error) of 0.035 (0.013) and 0.028 (0.010) on our patent stock and citation stock measures respectively.
significant additional information over and above raw patents counts.

Table 5.7: Market Value with Patents Measures.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$log(V_{i,t}/K_{i,t-1})$</td>
<td>1.620**</td>
<td></td>
<td>-0.352</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.537)</td>
<td></td>
<td>(0.828)</td>
<td></td>
</tr>
<tr>
<td>Patent Stock/capital</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>0.427***</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.147)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cites Stock/Capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.519**</td>
<td>0.491**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.221)</td>
<td>(0.243)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Year Cite Stock/Capital</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>No. Observations</td>
<td>2053</td>
<td>1748</td>
<td>1748</td>
<td>1748</td>
</tr>
<tr>
<td>No. Firms</td>
<td>205</td>
<td>182</td>
<td>182</td>
<td>182</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is $\log \ (\text{market value/ lagged capital})$. The estimation period covers 1969 until 1994 inclusive for column (1) and 1969 until 1990 inclusive for columns (2) to (4) which use the citation data which is only available for this shorted period. The *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors corrected for arbitrary heteroskedasticity.

In Table (5.8) we conduct some robustness tests on our basic models. In columns (1) and (2) we include both the patent stock and the lagged patent stocks measures. It is the lagged variable which is most informative in predicting productivity, suggesting that patented innovations take some time to enter the production function. In the market value equation, however, the current value of patents per unit capital has the larger coefficient (1.115) and is significant at the 15% level, while the lagged value has a coefficient of (0.661) and is not significant at all\textsuperscript{18}. This larger point estimate on the current value in the market value equation appears to reflect the forward looking

\textsuperscript{18}Although individually insignificant, the current and lagged values of they are jointly significant at the 1% level.
nature of the market value measure. In columns (3) and (4) we lag all our
right hand side variables one period to control for the possibility of endogeneity of our current explanatory variables. This does not noticeably change our results with significant effects of patents on productivity and market values. We also re-run this specification with all our explanatory variables lagged twice and again find our results looks very similar with a point estimate (standard error) of 0.042 (0.013) on patents in the productivity equation and of 1.01 (0.405) on (patents/capital) in the market value equation. We also look for both structural breaks and a time varying coefficient on our patent measures and find no significant evidence for either, with our main results remaining significant.
Table 5.8: Robustness Checks.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Sales</td>
<td>log(V_{i,t}/K_{i,t-1})</td>
<td>Real Sales</td>
<td>log(V_{i,t}/K_{i,t-1})</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Real Capital</td>
<td>0.468***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Log Real Capital</td>
<td></td>
<td>0.444***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.031)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Employment</td>
<td>0.541***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Log Employment</td>
<td></td>
<td>0.459***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.030)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Patent Stock</td>
<td>-0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Patents Stock</td>
<td>0.029**</td>
<td>0.036***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patent Stock/Capital</td>
<td>1.155</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.785)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Patent Stock/Capital</td>
<td>0.661</td>
<td>1.362**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.718)</td>
<td>(0.525)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>No. Observations</td>
<td>2053</td>
<td>1975</td>
<td>2053</td>
<td>1975</td>
</tr>
<tr>
<td>No. Firms</td>
<td>205</td>
<td>182</td>
<td>205</td>
<td>182</td>
</tr>
</tbody>
</table>

Notes: The dependent variable for the columns (1) and (3) is 'log real sales' and the dependent variable for columns (2) and (4) is 'log (market value/capital stock)' - both in 1985 prices. The estimation period covers 1968 until 1990 inclusive. The *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors corrected for arbitrary heteroskedasticity.

We also attempted to address any econometric concerns over the exogeneity of our knowledge stock measures by directly using instrumental variable estimators. However, with a total cross section of around 200 firms this could lead to serious small sample bias for a generalised method of moments type
estimator. On the other hand the Anderson-Hsiao type estimator can have a poor empirical performance because of the need to first difference the data which removes the levels information. This causes problems for instrumenting highly autoregressive series (like the patents stock) in a first differenced specification (see, for example, Bond and Blundell, 1998). For this reason previous studies using patenting data such as Hall et al. (2000) have focused on using OLS or within groups estimators. Because of these problems with IV estimation we only undertake some exploratory robustness checks here in addition to those in columns (3) and (4) of Table (5.8), using an estimator which is more appropriate in this long time series setting. This estimator uses the levels information from a within groups estimator but instruments with lagged explanatory variables to deal with any simultaneity problems, and has been used for example, in Mark, McGuire and Papke (2000) and Bloom, Griffith and Van Reenen (2001).

These instrumental variables point estimates are approximately similar to those delivered in tables (5.6) to (5.8) but with much larger variances. The point estimates (standard errors) on the patenting stock and citation weighting patenting stocks in the productivity equations are 0.048 (0.026)

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19 The GMM estimator is asymptotic in the cross section on a year by year basis so that it is the yearly number of firms not the total number which is important.

20 To be precise, the data is transformed into orthogonal deviations, where the orthogonal deviations transformation of $y_{i,t}$, labelled $y_{i,t}^*$, has the form (see Arrelano and Bond, 1991)

$$y_{i,t}^* = \left( y_{i,t} - \sum_{j=t+1}^{T} y_{i,j} \right) \left( \frac{T-t}{T-t+1} \right)^{\frac{1}{2}} \text{ for } t = 1, 2, ..., T - 1$$

The instruments for labour, capital and patents are labour, capital and patents lagged two and three years (also in orthogonal deviations). The instruments for (patents/capital) are also (patents/capital) lagged two and three years (in orthogonal deviations).
and 0.065 (0.035) respectively, which are larger than those in the comparable columns (4) and (5) of table (5.6), but are somewhat imprecisely estimated. For the market value specifications the point estimates (standard errors) on the patenting stock and citation weighted patenting stocks are 2.474 (0.948) and 0.792 (0.293) respectively, which are also somewhat larger than those presented in table (5.7) columns (1) and (2), although again quite imprecisely estimated. We concluded that, if anything, treating patents as exogenous causes an underestimation of their importance.

Finally, Table (5.9) reports our results from investigating the effects of uncertainty on the productivity response to patenting. In column 1 the patenting uncertainty interaction term takes the predicted negative sign in our productivity equation, and is significant at the 5% level. The coefficient on the level of uncertainty ($\sigma_i$), which is theoretically ambiguous, is also negative but not significant at the 5% level. When we move to the within groups specification in column (2) by including a full set of firm dummies we have to drop this firm specific uncertainty term $\sigma_i$ since this is collinear with the firm dummies. In column (2) we see that the patenting interaction term is, as before, negative and significant at the 1% level. The size of this interaction coefficient (-0.01) suggests that increasing a firm’s uncertainty by one standard deviation (0.42) from the median level of uncertainty (1.39) would reduce the elasticity of productivity with respect to patents from 0.024 to 0.199. Hence, for a one standard deviation increase in uncertainty the patenting effect on productivity falls by about 20%, a moderate but not enormous change.

In column (3) we investigate the levels and interaction effects of uncer-
tainty on the market value in an OLS equation. In line with our theoretical predictions the interaction effect on market values is less significant and of a smaller magnitude than the direct patenting effect. This is because market values are a forward looking measure and so will only incorporate the effects of higher uncertainty to the extent that it impact on total discounted cash flows. The levels effect of uncertainty, however, is highly significant and negative suggesting some direct effects of uncertainty on market values. This could possibly be through a real options effect, which is theoretically ambiguous, or through some other channel such as the greater discount rate associated with more uncertain shares\textsuperscript{21}. In column (4) we include a full set of firm dummies to control for fixed differences between firms. The uncertainty-patenting interaction term remains negative but is now insignificant at conventional levels suggesting only a limited effect of slower patent embodiment on the long term discounted cash flows.

\textsuperscript{21}Strictly speaking the relationship between uncertainty and capital valuation implied by theories such as the Capital Asset Pricing Model (CAPM) or the Consumption CAPM relies on covariance (with the market) rather than variance concepts. Since covariances and variances are likely to be positively linked, however, this negative statistical relationship between variance and capital valuation is not surprising.
Table 5.9: Real Options Effects of Uncertainty.

<table>
<thead>
<tr>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Sales</td>
<td>Real Sales</td>
<td>log(V_{i,t}/K_{i,t-1})</td>
<td>log(V_{i,t}/K_{i,t-1})</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Real Capital</td>
<td>0.451***</td>
<td>0.446***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.020)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Employment</td>
<td>0.517***</td>
<td>0.553***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Patent Stock</td>
<td>0.025**</td>
<td>0.038***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\sigma_i \times Log Patent Stock</td>
<td>-0.015**</td>
<td>-0.010***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patent Stock/Capital</td>
<td></td>
<td></td>
<td>0.913***</td>
<td>1.743***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.338)</td>
<td>(0.447)</td>
</tr>
<tr>
<td>\sigma_i \times Patent Stock/Capital</td>
<td>-0.265*</td>
<td>-0.073</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.159)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>\sigma_i</td>
<td>-0.036</td>
<td>-0.297***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.048)</td>
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</tr>
<tr>
<td>Firm Dummies</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Time Dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>No. Observations</td>
<td>2053</td>
<td>2053</td>
<td>2037</td>
<td>2037</td>
</tr>
<tr>
<td>No. Firms</td>
<td>211</td>
<td>211</td>
<td>205</td>
<td>205</td>
</tr>
</tbody>
</table>

Notes: The dependent variable for the first two columns is ‘log real sales’ and the dependent variable for the second two columns is ‘log real market value’ - both are in 1985 prices. The estimation period covers 1968 until 1990 inclusive. The *** denotes 1% significance, ** denotes 5% significance and * denotes 10% significance. Standard errors corrected for arbitrary heteroskedasticity.

To account for the possible effects of market-wide bubbles and fads we also calculate a second measure of uncertainty, using the variance of the firm’s daily share returns normalized by the return on the FTSE All-Share index. This measure eliminates common stock market volatility. Results using this normalized measure are almost identical to those reported in table (5.9), and are available on request from the authors.
5.6 Conclusions

Patents citations are a potentially powerful indicator of technological innovation. Our analysis of the new IFS-Leverhulme database on over 200 major British firms since 1968 has uncovered some interesting results. First, we show that patents have had an economically and statistically significant impact on firm-level productivity and market value. For example, a doubling of the citation-weighted patent stock increases total factor productivity by 3%. We find that citations are more informative than the simple patent counts that have been used previously in the literature. Secondly, we find that while patenting feeds into market values immediately it appears to have a slower effect on productivity. Thirdly, we find that higher market uncertainty, reduces the impact of new patents on productivity. This is consistent with a simple “real options” effect that has been found to be important in the literature on tangible investment.

There are several future directions to take this stream of research. We have not investigated the technological spillovers that have been a focus of attention in the recent literature. Patent citations are a potentially useful source of information in tracking the flows of knowledge across industries and countries and we intend to use the citations data in combination with R&D to investigate spillovers. A second area of interest is in probing the uncertainty results in more detail. If more uncertain environments reduce the productivity benefits from patents then it is likely that reductions in uncertainty (as is a focus of recent government policy to reduce “boom and bust”) will have effects on firms incentives to innovate. A natural extension of
this work is to augment the patenting equations with measures of uncertainty to uncover the importance of volatility in affecting innovation.
5.7 Appendices

1. Selecting the sample of UK firms

To obtain a manageable sample of firms for the matching stage we took UK firms from UK Datastream with names starting from A to L on which we also have company ownership data from the Leverhulme Company Ownership project. This set of firms was then supplemented by any other UK firms which we believed was likely to be a significant innovator, as proxied by their appearance in the top 100 R&D spenders in the “UK R&D Scoreboard” (DTI, 1997). This resulted in a final sample of 415 firms against which we attempted to match patenting data.

The ISIC breakdown of the selected firms is given in Table (5.10) below where it can be seen that we have a mix of various sectors, but with a concentration in the traditionally innovative chemicals, pharmaceuticals and engineering sectors.

2. Matching Up the Patent Data

The main patent data set contains information on over 6 million patents granted between 1968 and 1996 by the United States Patent Office. This information includes indicators on the name of the inventor, the location at which the patent was taken out and the patents which the patent cites itself. These patents also have an assignee code which is an indicator matching up the patent with the organisation which registered it, and there are about 140,000 different patenting assignees. Since firms register patents under a

22See, for example, Bond and Chennells (2000), for more details.
Table 5.10: Industry Breakdown of Patenting Firms.

<table>
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<tr>
<th>ISIC</th>
<th>Industry</th>
<th>No. observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3100</td>
<td>Food and Beverages</td>
<td>216</td>
</tr>
<tr>
<td>3200</td>
<td>Textiles and Apparel</td>
<td>55</td>
</tr>
<tr>
<td>3400</td>
<td>Paper and Paper Products</td>
<td>49</td>
</tr>
<tr>
<td>3500</td>
<td>Chemicals &amp; Pharmaceuticals</td>
<td>409</td>
</tr>
<tr>
<td>3600</td>
<td>Non-Metallic Minerals</td>
<td>143</td>
</tr>
<tr>
<td>3700</td>
<td>Basic Metal Industries</td>
<td>93</td>
</tr>
<tr>
<td>3800</td>
<td>Engineering &amp; Metal Products</td>
<td>793</td>
</tr>
<tr>
<td>3900</td>
<td>Other Manufacturing</td>
<td>111</td>
</tr>
<tr>
<td>4000</td>
<td>Electricity, Gas and Water</td>
<td>10</td>
</tr>
<tr>
<td>5000</td>
<td>Construction</td>
<td>20</td>
</tr>
<tr>
<td>689</td>
<td>Other Services</td>
<td>267</td>
</tr>
</tbody>
</table>

A variety of different assignee names, usually the parent name of the firm or one of their subsidiaries, it is not possible to directly match up these assignees with the ultimate parent. For example, Glaxo PLC. has 354 patents listed directly under the assignee "GLAXO GROUP LIMITED", 196 patents listed under the assignee "GLAXO LABORATORIES LIMITED", and a further 80 patents listed under an assortment of other assignee names such as "GLAXO INC" and "GLAXO CANADA". Whilst the linkage between the Glaxo parent group and its subsidiaries are obvious in this case due to the common parent firms name in many other cases this link is less clear. For example, one of the largest patenting firms in our group is "BTR PLC" whose three largest patenting assignees are called "DUNLOP LTD", "STEWART-WARNER CORPORATION" and "BTR INDUSTRIES", where only the third assignee would show up under a direct computerised name search. Because of the inadequacy of direct computerised name matching we had to undertake a careful two stage process to try and match up all the assignees.
of our ultimate parent list of UK quoted firms to our list of 140,000 patenting assignees. This was carried as follows:

**Stage 1**

We selected the larger assignees - deemed to be those with 10 or more patents - which numbered about 12,000 out of the initial list of all 140,000 assignees and accounted for 5.2 million (or 87%) of all of the 6 million patents and undertook a manual match against these. This was an extremely lengthy procedure which involved individually looking up each of these 12,000 assignees by turn in "Who Owns Whom 1985" to check whether its ultimate parent was a UK company. If the ultimate parent was a UK company we then name checked this against our sample of 415 quoted UK companies to see whether it was owned by a firm in this group. If the assignees was owned by one of our 415 quoted UK companies we then typed in the appropriate Dscode matching details.

**Stage 2**

For the remaining 128,000 assignees which had registered less than 10 patents (and accounted for only 0.8 million patents) we had to rely on direct computerised name matching by searching on key string words in the ultimate parent name and then cross checking with "Who Owns Whom" to ensure this was a correct match. For example, string searching on Glaxo welcome revealed two additional patenting assignees with less than 10 patents which would have been missed in the first stage of matching- "GLAXO CANADA, INC" with 2 patents and "GLAXO OPERATIONS UK LIMITED" with 3 patents. Whilst this procedure is less desirable (we proportionally matched
only a third as many assignees by computer compared to by hand) it was the only feasible matching procedure. Since only 13% of the total patents stock number is contained in this longer list of small firms the degree of omitted patenting should anyway not be great.

When this matching process was complete we aggregated the patenting information assignees up to their common parent company to yield firm level patent statistics.

3. Extending the employment data series

There are two main problems with the UK employment series. First, employment information was only recorded for a sub-sample of Datastream firms prior to 1982. For those firms where data was missing we matched in the information from the EXSTAT database of company accounts which contains employment data back to 1972. Secondly, prior to 1982 UK companies were required to report their total UK labour force, but from 1982 onwards have been required to report their global labour force, leading to a break in the series. Some firms report the global employment series all the way through, but many report only UK employment before 1982. Since we have a long time series of patenting data pre 1982 we have tried to bridge across this gap rather than drop the earlier data. To bridge this gap we have use two methods. For firms who report both UK and global labour force figures post 1982 we use the post-1982 UK/global ratio to extrapolate the pre-1982 global labour force based on the reported UK labour force. For the remaining firms we assume that the growth in total labour force between 1981 and 1982 is equal to the average of the growth rates two years before and two years
after this split. This enables us to generate a forecasted 1982 UK labour force figure, from which we can obtain an estimated UK/global labour force split and extrapolate the global labour force pre-1982 using the reported UK labour force. Since the mean (median) share of global employment based in the UK in 1982 is 0.87 based on those firms which report both figures\textsuperscript{23} the errors arising from trying to extrapolate the global labour force from the UK labor force exercise should not be great. However, to check the robustness of our results to this we re-estimated all our regressions including a dummy variable for extrapolated labour data and found no significant change in the estimated results.

\textsuperscript{23}Because the more internationalised firms are more likely to continue reporting this UK/global employment split this may actually underestimate the share of global employment based in the UK, so that its true average is probably greater than 0.87.


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CABALLERO, R., ENGEL, E. and HALTIWANGER, J. (1997), "Ag-


PINDYCK, R. (1988), "Irreversible Investment, Capacity Choice, and the


