Online Appendices: Not Intended for Publication Unless Requested

8 Appendix A - Theoretical Details and Proofs

In this appendix we give more technical details and proofs of our main results. There are multiple equivalent conventions which can be used in the decentralization of this economy. In the text when describing the intermediate goods firms within the North and South, we found it convenient to treat these firms as each having identical technologies but devoted to the production of consumption goods, innovation of new goods, or production of existing goods. In that structure, although we can for convenience speak of a flow Y_t of output, there is no formal "final good" or "final goods firm," and intermediate goods firms directly demand human capital from households and intermediate goods from other firms. However, please note that in the Appendices of the paper we have used an alternative formulation, one which is equivalent in its allocations. In the alternative Appendix formulation which is used in the definitions and proofs below, we speak of a final goods firm operating under perfect competition, which creates a physical flow of final goods output that a single class of intermediate goods firms, which are equity-financed, must direct optimally towards production and innovation in the interest of their owners. Although equivalent in terms of allocations, these formulations do involve different notation.

Definition 1 *Closed-Economy Equilibrium*

Given initial conditions A_0, x_{j0}, an equilibrium is a path of wages, interest rates, stock prices, and intermediate goods prices w_t, r_t, q_{ft}, p_{jt}, together with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods dividends, aggregate innovation quantities, firm variety portfolios, and aggregate variety quantities s_{ft}, b_t, H_t^D, x_{jt}^D, x_{jt+1}^S, M_{ft+1}, d_{ft}, A_t, $A_{ft,}$ $M_{t,}$ such that

Households Optimize: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t,b_{t+1},s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1}+C_t+\sum_{f=1}^{N} q_{ft}(s_{ft}-s_{ft-1}) \leq (1+r_{t+1})b_t+w_t H+\sum_{f=1}^{N} d_{ft}s_{ft}.$$

Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm statically optimizes profits by choosing labor demand $H_t^D$and intermediate goods input demands x_{jt}^D,i.e. these decisions solve

$$\max_{H_t,x_{kt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha}\, dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Intermediate Goods Firms Optimize: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices p_{jt}, $j \leq A_{t-1}$, and aggregate variety and innovation levels A_t, M_{t+1} as given, intermediate goods firms maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods for use next period x^S_{jt+1},and the price of on-patent intermediate goods $p_{jt}, j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} x_{jt+1} dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt} x_{jt} dj$$

$$Z_{ft} = \nu M^{\gamma}_{ft+1} A^{1-\gamma}_t$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear:

$$H^D_t = H,\ b_{t+1} = 0,\ s_{ft} = 1,\ x^D_{jt+1} = x^S_{jt+1}$$

Final Goods Market Clears:

$$Y_t = C_t + \int\limits_0^{A_{t+1}} x_{jt+1} dj + \sum_{f=1}^{N} Z_{ft}$$

Innovation and Variety Consistency Conditions Hold:

$$A_{t+1} = A_t + M_{t+1},\ A_{ft+1} = A_{ft} + M_{ft+1},\ M_{t+1} = \sum_{f=1}^{N} M_{ft+1},\ A_t = \sum_{f=1}^{N} A_{ft.}$$

Definition 2 *Open-Economy Equilibrium*

Given any initial conditions A_0, x_{j0}, x^*_{j0}, along with a sequence of trade restrictions ϕ_t, an equilibrium in the open economy is a set of terms of trade, interest rates, wages, stock prices, and intermediate goods prices q_t , r_t , r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft} ,p_{jt}, and p^*_{jt} , along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}, x^S_{jt+1}, x^{*S}_{jt+1}, M_{ft+1}, A_{jt}, A^*_{ft}, d_{ft}, d^*_{ft}, M_t, I_t, R_t$, and A_t such that

Northern Household Optimizes: Taking wages w_t , interest rates r_t , and stock prices q_{ft} as given, the representative household in the North maximizes the present discounted value of its consumption stream by choosing period consumption C_t , debt b_{t+1} , and share purchases s_{ft} , i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft} \left(s_{ft} - s_{ft-1}\right) \leq (1 + r_{t+1}) b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft} .$$

Southern Household Optimizes: Taking wages w_t^* , interest rates r_t^* , and stock prices q_{ft}^* as given, the representative household in the South maximizes the present discounted value of its consumption stream by choosing period consumption C_t^* , debt b_{t+1}^* , and share purchases s_{ft}^* , i.e. these decisions solve

$$\max_{C_t^*, b_{t+1}^*, s_{ft}^*} \sum_{t=0}^{\infty} \frac{\beta^t \left(C_t^*\right)^{1-\sigma}}{1-\sigma}$$

$$b_{t+1}^* + C_t^* + \sum_{f=1}^{N} q_{ft}^* \left(s_{ft}^* - s_{ft-1}^*\right) \leq (1 + r_{t+1}^*) b_t^* + w_t^* H^* + \sum_{f=1}^{N} d_{ft}^* s_{ft}^* .$$

Northern Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the North statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} \, dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} \, dj.$$

Southern Final Goods Firm Optimizes: Taking wages w_t^* and intermediate goods prices p_{jt}^* as given, the competitive representative final goods firm in the South statically optimizes profits by choosing labor demand H_t^{*D} and intermediate goods input demands x_{jt}^{D*}, i.e. these decisions solve

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^{\alpha} \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} \, dj - w_t^* H_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj .$$

Northern Intermediate Goods Firm Optimizes: Taking marginal utilities m_t , perfectly competitive off-patent intermediate goods prices $p_{jt}, j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t , and M_{t+1} as given, intermediate goods firms f in the North maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x^S_{jt+1}, x^{*S}_{jt+1}, and the price of on-patent intermediate goods p_{jt}, $j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m_t \, d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt} \, (x_{jt} + x^*_{jt})dj$$

$$Z_{ft} = \nu M^{\gamma}_{ft+1} A^{1-\gamma}_t$$

Southern Intermediate Goods Firm Optimizes: Taking marginal utilities m^*_t and perfectly competitive off-patent intermediate goods prices p^*_{jt}, $j \leq A_{t-1}$ as given, intermediate goods firms f in the South maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A^*_{ft} for use next period x^S_{jt+1}, x^{*S}_{jt+1}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m^*_t \, d_{ft}$$

$$d^*_{ft} + \int\limits_{A^*_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj \leq \int\limits_{A^*_{ft}} p^*_{jt}(x_{jt} + x^*_{jt})dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$H^D_t = H, \; H^{*D}_t = H^*,$$

$$b_{t+1} = 0, \; b^*_{t+1} = 0,$$

$$s_{ft} = 1, \; s^*_{ft} = 1,$$

$$x^D_{jt} = x^S_{jt}, \; x^{*D}_{jt} = x^{*S}_{jt}.$$

Final Goods Markets Clear

$$Y_t = H^{\alpha} \int x^{1-\alpha}_{jt} dj = C_t + R_{t+1} x_{Rt+1} + M_{t+1}(x_{Mt+1} + x^*_{Mt+1}) + \sum_{f=1}^{N} Z_{ft}$$

$$Y^*_t = (H^*)^{\alpha} \int_0^{A_t} \left(x^*_{jt}\right)^{1-\alpha} dj = C^*_t + R_{t+1} x^*_{Rt+1} + I_{t+1}(x_{It+1} + x^*_{It+1})$$

No Arbitrage Pricing Condition Holds

$$p_{jt} = q_t p^*_{jt}$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$$

Innovation and Variety Consistency Conditions Hold:

$$\phi_t \ (R_t + I_t) = I_t, \ I_t + R_t = A_{t-1}, \ I_t + R_t + M_t = A_t,$$

$$A_{ft+1} = A_{ft} + M_{ft+1}, \ M_t = \sum_{f=1}^{N} M_{ft}, \ M_t + R_t = \sum_{f=1}^{N} A_{ft}, \ I_t + R_t = \sum_{f=1}^{N} A^*_{ft}.$$

Southern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Southern economy only.

Although the fully mobile economy with a trade shock has essentially the same equilibrium concept as laid out in the previous section initially discussing the open economy, we must be more explicit about the trapped factors environment. In the trapped factors equilibrium, Northern intermediate goods firms face an additional constraint due to the adjustment costs preventing them from immediately responding in their input usage to the new trade shock. Formally, they must solve the modified problem

$$\max_{p_{ft}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}, X_{ft}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj$$

$$\int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \geq X_{ft} \left(\phi^E_{t,t+1}\right),$$

where $X_{ft}\left(\phi^E_{t,t+1}\right)$ is the optimal input demand for period t, given expectations of the trade restriction $\phi^E_{t,t+1}$ for the next period. X_{ft} is also indexed by f and depends both upon the number of M goods that the firm plans to produce for next period, as well as the number of R goods that the firm has in its portfolio and plans to produce for the next period. Therefore, although these portfolio shares are only allocative in a period in which a trade shock occurs, we must be explicit about the structure we assume for the pre-shock portfolios of R goods held by each firm f, as well as the actual allocation of the trade shock liberalization among existing firms' measures of R goods. We now define some additional notation. Let $\widetilde{s}_f$ be the share of off-patent R goods production firm f anticipates doing before the trade shock, where $\sum_{f=1}^{N} \widetilde{s}_f = 1$. Then, let the trade shock allocate destruction of R goods production opportunities across firms so that only the proportion χ_f of R goods varieties can still be produced in each firm. As long as we have the consistency condition

$$\sum_{f=1}^{N} \widetilde{s}_f \chi_f \left(1-\phi\right) A_t = (1-\phi')A_t,$$

an arbitrary choice of χ_f will be consistent with the trade shock $\phi \to \phi'$. We will henceforth make the assumption that $\widetilde{s}_f = \frac{1}{N}$ for all firms, i.e. that pre-shock allocations of R goods production is uniform across firms. This assumption grows naturally out of our structure in which we assume that firms continue to be the producers of goods which they invented, even after these goods fall off-patent and become perfectly competitive. We also will now assume that N is even, and that half of the firms in the economy are in the "No Shock" industry, industry 1. The other half of firms in the economy, those in the "Shocked" industry 2, experience a loss of R goods production opportunities during the trade shock with only a fixed proportion χ_2 of R goods remaining. This framework is a rough approximation of the heterogeneity in the direct effects on firms in developed countries during the trade liberalizations of the early 2000s. Seen in this light, industries such as textiles which experienced a substantial loss of protection against manufacturers in low-wage economies such as China, can be identified with industry 2, while other industries would be represented by firms in group 1 in our environment. We now define a trapped factors equilibrium formally.

Definition 3 *Trapped Factors Trade Shock Equilibrium*

Given any initial conditions A_0, $x_{j0,}$ x^*_{j0} and a sequence of trade restrictions

$$\phi_s = \begin{cases} \phi, s \le t, \\ \phi', s > t \end{cases},$$

where the trade shift from ϕ to $\phi' > \phi$ is unanticipated and affects only Shocked industry 2, leaving the proportion χ_2 of R goods in industry 2 restricted, a trapped factors equilibrium in the open economy is a set of terms of trade, interest rates, wages, stock prices, and intermediate goods prices q_t, r_t, r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft}, p_{jt},and p^*_{jt}, along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}, x^S_{jt+1}, x^{*S}_{jt+1}$, M_{ft+1}, A_{ft},A^*_{ft},d_{ft}, d^*_{ft},$M_{t,}I_t$, R_t, and A_t such that

Northern Household Optimizes: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household in the North maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{ft}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft}(s_{ft} - s_{ft-1}) \leq (1 + r_{t+1})b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft}.$$

Southern Household Optimizes: Taking wages w_t^*, interest rates r_t^*, and stock prices q_{ft}^* as given, the representative household in the South maximizes the present discounted value of its consumption stream by choosing period consumption C_t^*, debt b_{t+1}^*, and share purchases s_{ft}^*, i.e. these decisions solve

$$\max_{C_t^*, b_{t+1}^*, s_{ft}^*} \sum_{t=0}^{\infty} \frac{\beta^t \left(C_t^*\right)^{1-\sigma}}{1-\sigma}$$

$$b_{t+1}^* + C_t^* + \sum_{f=1}^{N} q_{ft}^*(s_{ft}^* - s_{ft-1}^*) \leq (1 + r_{t+1}^*)b_t^* + w_t^* H^* + \sum_{f=1}^{N} d_{ft}^* s_{ft}^*.$$

Northern Final Goods Firm Optimizes: Taking wages $w_t$and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the North statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^\alpha \int_0^{A_t} (x_{jt})^{1-\alpha} dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Southern Final Goods Firm Optimizes: Taking wages w_t^* and intermediate goods prices p_{jt}^* as given, the competitive representative final goods firm in the South statically optimizes profits by choosing labor demand H_t^{*D} and intermediate goods input demands x_{jt}^{D*}, i.e. these decisions solve

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^\alpha \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} dj - w_t^* H_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj.$$

Northern Intermediate Goods Firm Optimizes: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices p_{jt}, $j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t, M_{t+1} as given intermediate goods firms in the North maximize firm value, the discounted stream of dividends, by first choosing the quantity of inputs $X_{ft}\left(\phi_{t,t+1}^E\right)$ given their expectations of trade policy next period, then choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x_{jt+1}^S, x_{jt+1}^{*S}, and the price of on-patent intermediate goods p_{jt}, $j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x_{jt+1}^*, X_{ft}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt})dj$$

$$\int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \geq X_{ft}\left(\phi^E_{t,t+1}\right)$$

$$Z_{ft} = \nu M^{\gamma}_{ft+1} A^{1-\gamma}_t$$

where we have that

$$\phi^E_{s,s+1} = \begin{cases} \phi, & s \leq t \\ \phi', & s > t \end{cases}.$$

Southern Intermediate Goods Firm Optimizes: Taking marginal utilities m^*_t and perfectly competitive off-patent intermediate goods prices $p^*_{jt}, j \leq A_{t-1}$ as given, intermediate goods firms in the South maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A^*_{ft} for use next period x^S_{jt+1},x^{*S}_{jt+1}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m^*_t d_{ft}$$

$$d^*_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj \leq \int\limits_{A_{ft}} p^*_{jt}(x_{jt} + x^*_{jt})dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$H^D_t = H,\ H^{*D}_t = H^*,$$
$$b_{t+1} = 0,\ b^*_{t+1} = 0,$$
$$s_{ft} = 1,\ s^*_{ft} = 1,$$
$$x^D_{jt} = x^S_{jt},\ x^{*D}_{jt} = x^{*S}_{jt}.$$

Final Goods Markets Clear:

$$Y_t = H^{\alpha} \int x^{1-\alpha}_{jt} dj = C_t + \int_{R_{t+1}} x_{jt+1} dj + \int_{M_{t+1}} (x_{jt+1} + x^*_{jt+1})dj + \sum_{f=1}^{N} Z_{ft}$$

$$Y^*_t = (H^*)^{\alpha} \int\limits_0^{A_t} \left(x^*_{jt}\right)^{1-\alpha} dj = C^*_t + \int_{R_{t+1}} x^*_{jt+1} dj + \int_{I_{t+1}} (x_{jt+1} + x^*_{jt+1})dj$$

No Arbitrage Pricing Condition Holds

$$p_{kt} = q_t p^*_{jt}$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$$

Innovation and Variety Consistency Conditions Hold:

$$_{\phi_t}(R_t+I_t) = I_t, I_t+R_t= A_{t-1}, I_t+R_t+M_t= A_t,$$

$$A_{ft+1}= A_{ft}+M_{ft+1},\ M_t=\sum_{f=1}^{N}M_{ft},\ M_t+R_t=\sum_{f=1}^{N}A_{ft},\ I_t+R_t=\sum_{f=1}^{N}A^*_{ft}.$$

Southern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Southern economy only.

Proof of Proposition 1: Closed Economy Balanced Growth Path To complete the proof of Proposition 1, we need to show that the rates of growth of output, consumption, and varieties are equal on the balanced growth path. The final goods market clearing condition is

$$C_t = H^{\alpha}\left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-a} + I_t x_{It}^{1-\alpha}\right] - M_{t+1}x_{Mt+1} - R_{t+1}x_{Rt+1} - \sum_{f=1}^{N} Z_{ft},$$

where we note that since it is the measure of off-patent varieties, $R_t = A_{t-1}$, and the measure of innovated varieties $M_t = gA_{t-1}$. Now, recall the assumption of balanced growth. If we define the growth rate of consumption by g_C, and note that the by symmetry the individual firm patenting ratios $g^f = \frac{g}{n}$, we can use the intermediate goods firm pricing rules to rewrite the final goods market clearing condition as

$$\frac{C_t}{A_t} = \frac{1}{1+g}H\left[(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left((1-\alpha)^{\frac{1-\alpha}{\alpha}}+1\right)\beta^{\frac{1-\alpha}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}\right] - g(1-\alpha)^{\frac{2}{\alpha}}\beta^{\frac{1}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}H$$
$$-(1-\alpha)^{\frac{1}{\alpha}}\beta^{\frac{1}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}H - N\nu\left(\frac{g}{N}\right)^{\gamma}.$$

Since $\frac{C_t}{A_t}$ is constant, we conclude that $g = g_C$, so that the innovation optimality condition reads

$$\frac{\nu\gamma}{N^{(\gamma-1)}}g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}H.$$

This expression motivates the choice of the scaling constant

$$\nu = \frac{N^{(\gamma-1)}}{\gamma},$$

so that the balanced growth path growth rates are invariant to the number of firms or the degree of cost externalities across firms as well as the number of firms N. We obtain the balanced growth path innovation optimality condition

$$g^{\gamma-1}= \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}H.$$

The left-hand side, the marginal cost of innovation, is strictly increasing in g, is equal to 0 when $g = 0$, and limits to ∞ as $g \to \infty$. The right-hand side, the discounted monopoly profits from innovation, is strictly decreasing in g, is equal to $\Omega\beta^{\frac{1}{\alpha}}H > 0$ when $g = 0$, and limits to 0 as $g \to \infty$. We conclude that a balanced growth path equilibrium exists and is uniquely determined by the value of g which satisfies the innovation optimality condition. This completes the proof.

Proof of Proposition 2: Open Economy Balanced Growth Path The demand schedules for intermediate goods, based on the Northern and Southern final goods firms' technologies, are given by

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}$$
$$x_{jt}^* = (1-\alpha)^{\frac{1}{\alpha}} H^* \left(p_{jt}^*\right)^{-\frac{1}{\alpha}},$$

where $p_{jt}$and $p_{jt}^*$are the prices of intermediate good variety j in Northern and Southern output units, respectively, and $p_{jt} = q_t p_{jt}^*$. The optimality conditions for the Northern intermediate goods firm, combined with the Euler equations of the Northern representative household for debt and equity, are given by

$$p_{Rt+1} = 1 + r_{t+1}$$
$$p_{Mt+1} = \frac{1 + r_{t+1}}{1-\alpha}$$
$$\frac{\partial}{\partial M_{ft+1}} Z_{ft+1} = \left(\frac{1}{1+r_{t+1}} p_{Mt+1} - 1\right)(x_{Mt+1} + x_{Mt+1}^*).$$

Differentiating the cost function and substituting in the optimal pricing rules we have that the third condition, the innovation optimality condition, is given by

$$\nu\gamma(g_{t+1}^f)^{(\gamma-1)} = \Omega\beta^{\frac{1}{a}}(\frac{C_{t+1}}{C_t})^{-\frac{\sigma}{\alpha}}(H + q_{t+1}^{\frac{1}{\alpha}} H^*).$$

Now the balanced trade condition can be written

$$M_t p_{Mt} x_{Mt}^* = I_t p_{It} x_{It}$$
$$g_t A_{t-1} \frac{(1+r_t)}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} H^* \left(\frac{(1+r_t)}{q_t(1-\alpha)}\right)^{-\frac{1}{\alpha}} = \phi A_{t-1} q_t (1+r_t^*)(1-\alpha)^{\frac{1}{\alpha}} \left(q_t(1-\alpha)\right)^{-\frac{1}{\alpha}} H$$
$$q_t = \left(\frac{\phi H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}},$$

where $\Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$. Now, applying the assumption of balanced growth, we immediately obtain from the Euler equations of both representative households that interest rates in the Northern and Southern economies, as well as the terms of trade, are constant. Also, exactly as in the proof of Proposition 1, the final goods market clearing conditions for each economy, together with the assumption of balanced growth, imply that the ratios

$$\frac{C_t}{A_t}, \frac{C_t^*}{A_t}$$

are constant, so that we conclude that

$$(1+r) = (1+r^*) = \beta^{-1}(1+g)^{\sigma}.$$

Using this, we conclude that

$$q = \left(\frac{\phi H}{gH^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi.$$

Now the innovation optimality condition can be rewritten as

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}(H + q^{\frac{1}{\alpha}}H^*).$$

Also, substituting the terms of trade formula/balanced trade condition into the innovation optimality condition yields

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}\left(H + \left(\frac{\phi H}{gH^*}\right)^{\frac{1}{2-\alpha}} \Psi^{\frac{1}{\alpha}} H^*\right).$$

As a function of g, the marginal cost of innovation on the left-hand side is strictly increasing in g, starting at 0 and growing exponentially to ∞ as $g \to \infty$. The right-hand side, the discounted monopoly profits from sale of newly patented goods in the North and the South, is strictly decreasing in g, asymptoting to ∞ as $g \to 0$ and to 0 as $g \to \infty$. We conclude both that there exists a balanced growth path equilibrium for this economy, and that it is the unique balanced growth path growth rate. For any given fixed value of ϕ, we denote this growth rate, and the associated terms of trade, by $g(\phi)$ and $q(\phi)$. This completes the proof.

Appendix B - Parameter Values and Robustness Checks

Calculating the ratio of H to H^*

To calculate the ratio of H to H^*, we follow the human capital accounting approach in Hall and Jones (1999) and compute the human capital endowment in country c from the Barro and Lee (2010) data as $H_c = e^{\mu_c S_c} P_c$, where S_c is the average number of years of schooling completed in the adult population above age 25, and P_c is the size of the population of the country c in 2000. We take into account the differences in educational quality and the returns to schooling across countries by using the Mincerian returns to education of immigrants in the United States from country c, μ_c, from Table 4in Schoellman (2011). If Mincerian returns for a country c is not available in Schoellman (2011), we take $\mu_c = 7\%$ for non-OECD countries and $\mu_c = 9\%$ for OECD countries. These are the averages of returns to schooling for the two categories in Schoellman's sample. We finally define $H_{non-OECD} = 2.1 \sum_{c \notin OECD} H_c$, where the ratio 2.1 corrects for the fact that not all non-OECD countries are represented in the Barro and Lee data. In particular 2.1is equal to the ratio of the non-OECD to OECD population ratio in 2000 in the Wolfram Alpha database (with full global coverage) to the non-OECD to OECD population ratio in 2000 in the Barro and Lee data. Such a procedure relies on the implicit assumption that the schooling rates and returns to education in countries not represented in the Barro and Lee data are similar to those with data present. From the procedure above we obtain $\frac{H^*}{H} \approx 2.96$, which we round to 3.0 in the text discussion.

Computing Patent Ratios

United States Patent and Trademark Office data on patents granted from 1977-2006, by application year and nationality of assignee, are downloaded from the NBER website for the Patent Data Project, as of early 2013. This website represents an update of the data which was originally collected and documented in Hall, Jaffe, Trajtenberg (2001). Patents granted to multiple assignees are counted only once, and the nationality of the patent is determined by the first assignee. OECD status is as of application year. Total foreign, non-OECD, and Chinese patent ratios are equal to the number of granted patents with a particular application year, normalized by the total number of granted patents in the same application year. This normalization incorporates the reduction in grant numbers as the application year approaches the end of the sample, the well known application lag/truncation problem with patent data of this form. Figure B1 plots the proportion of all US patents granted by application year from any foreign nation, from non-OECD countries, and from Chinese assignees, for the years 1977-2006.

Calculating the Trade Shares

The real per capita output growth rate is from the US NIPA tables, computed as the average annual real GDP per capita growth rate from 1960-2010. Trade data was downloaded from the OECD-STAN database, and OECD GDP data comes from the Penn World Tables, Version 7.1. The non-OECD country to OECD imports to OECD output ratios were computed over the years 1997-2006. The period was chosen to incorporate the accession of China to the WTO in 2001, and the 10-year window accords with the model calibration of a period to 10 years. All of the data and simple calculations performed in the calibration procedure are available on Nicholas Bloom's website: http://www.stanford.edu/nbloom. Figure B2 plots the non-OECD imports to OECD GDP ratio over this period, together with Chinese imports into the OECD.

Trade policy substitution in the counterfactual away from China towards the rest of the non-OECD

Total observed low-wage import growth into the OECD as a share of GDP from 1997-2006 is equal to 3.1%. Growth in Chinese import shares was equal to 1.61%, implying that non-China/non-OECD countries saw their import shares into the OECD increase by 1.49%. The no China counterfactual in the main text assumed that the growth in Chinese import shares was completely removed from liberalization over this period. If, however, policy-makers partially substituted towards other non-OECD imports in lieu of Chinese imports, we would still see import share growth in the counterfactual. To analyze the quantitative magnitude of this substitution effect, we consider a case where exactly one half of Chinese import growth is realized in the no China counterfactual, via substitution towards other non-OECD countries. Starting with a low-wage import share of 3.9%, this "half substitution" case exhibits import share growth of 0.5*1.61+1.49 = 2.295%, so that the resulting target import to output ratio post-liberalization in the counterfactual is 3.9+2.295 = 6.195%. Figure B3 plots the resulting two trapped factors transition paths, analogous to Figure 7, in the total observed import liberalization and "Half China" cases. As can be seen immediately, the transition paths differ by less than the case in which all Chinese import growth is removed-, which works to reduce the marginal contribution of China to welfare to a total of 3.3% (North) and 3.2% (South). In this alternative counterfactual, the impact of China is equal to 20% (North) and 21% (South) of the overall welfare gains from trade observed in the data.

Other Robustness Checks

In this section we provide the main numbers underlying the robustness checks underlying Figure 8 in the main text. In particular, beginning from our baseline calibration, in Table $B1$ we list the post-shock balanced growth path growth rate, as well as the maximum growth rate along the trapped-factors transition path, for a number of alternative parameter choices.

Table B1: Growth Rate Robustness Checks

Parameter	Peak Transition Path Growth (%)	Post-Shock Balanced Growth (%)
$\beta = 1/1.04$	2.71	2.37
$\beta = 1/1.01$	2.74	2.37
$\eta = 0.5$	2.59	2.37
$\sigma = 2.0$	3.04	2.32
$\sigma = 1.5$	2.89	2.34
$\rho = 0.6$	2.84	2.46
$\rho = 0.4$	2.61	2.29
$\alpha = 0.5$	2.74	2.32
$\alpha = 0.7$	2.73	2.38
Baseline	2.73	2.37

Note: The first column records the parameter varied from our baseline calibration. The second column represents the maximum annualized percentage variety growth rate over the trapped factors transition path in the alternative calibrations. The third column represents the post-shock balanced growth path annualized percentage growth rate associated with the alternative calibration. The baseline calibration features parameter choices of $\rho = 0.5$, $\alpha = 0.667$, $\beta = 1/1.02$, $\sigma = 1.0$, and $\eta = 1.0$.

Also, note that in the text we mention an alternative calibration strategy for the pre-shock balanced growth path growth rate. If we compute the United States per capita real GDP growth rate over the period $1960 - 2001$ rather than the baseline calibration window of $1960 - 2010$, we obtain a pre-shock balanced growth rate of 2.3% rather than the baseline 2.0%. However, in this case, the peak transition path growth rate is 3.09%, and the post-shock balanced growth rate is 2.70%. Given the higher initial condition, this is almost a direct translation upwards of the baseline transition path. Given the nonlinearity of the model, such a result is not automatic.

Note that a previous version of our calibration strategy, with results published in "A Trapped Factors Model of Innovation," (*American Economic Review: Papers and Proceedings,* 2013) yielded smaller dynamic impacts of trade liberalization. Our improved calibration strategy here differs from that earlier work in three respects. First, we consider a model period of ten years rather than one year to match a more plausible effective monopoly length. Second, we base the calibration on imports to value added ratios rather than imports to gross output ratios, since data availability for China is better for value added. Third, instead of calibrating the post-liberalization trade openness via a "limiting" highest ϕ' which still maintained product-cycle trade (i.e. $q(\phi') < 1$), the first two calibration changes allow us to now directly match observed pre- and post-liberalization trade ratios in 1997 and 2006, which results in larger growth impacts more aligned with observed trade liberalization.

Appendix C - Solution Technique and Equilibrium Conditions for the Calibration

Please find both replication data files for the calibration exercise, as well as code to duplicate all of the quantitative results in the paper, on Nicholas Bloom's website at http://www.stanford.edu/nbloom/. We solve each of the systems of nonlinear equations laid out below using particle swarm optimization as implemented in R. This is an extremely robust global nonlinear optimization technique, and all solutions are computed with a summed squared percentage error across all equations of less than 10^{-7}.

Balanced Growth Path

As documented in the proof of Proposition 2, the balanced growth path growth rate $g(\phi)$ of the open economy given trade restriction ϕ is fully characterized by the equilibrium innovation optimality condition

$$g(\phi)^{\gamma-1}= \Omega\beta^{\frac{1}{a}}(1+g(\phi))^{-\frac{\sigma}{\alpha}}\left(H+\left(\frac{\phi H}{g(\phi)H^*}\right)^{\frac{1}{2-\alpha}}\Psi^{\frac{1}{\alpha}}H^*\right).$$

All other long-run quantities, in particular the interest rates and exchange rate, are direct functions of this balanced growth path growth rate through the Euler equations and balanced trade condition

$$(1+r(\phi)) = (1+r^*(\phi)) = \beta^{-1}(1+g(\phi))^{\sigma}$$

$$q(\phi) = \left(\frac{\phi H}{g(\phi)H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi.$$

Fully Mobile Transition Dynamics

To compute the transition dynamics of the fully mobile model in response to a trade shock in period 0, starting from the balanced growth path associated with trade restriction ϕ, we first pick a horizon T. We also normalize $A_0 = 1$. Then, we assume that the model has converged to the balanced growth path associated with $\phi'$by period T. This structure requires that we solve for $3(T-1)$prices, $\{q_t, r_t, r_t^*\}_{t=2}^{T}$. These $3(T-1)$ prices are pinned down by $3(T-1)$ equations: the balanced trade condition, the Northern Euler equation, and the Southern Euler equation, in periods $1, ..., T-1$. These equations are given by

$$q_t= \left(\frac{\phi H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi\left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}},$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_{t+1}),$$

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} = \beta(1+r_{t+1}^*).$$

We note that all allocations in the transition path are a function of these three prices. Intermediate goods prices follow the monopoly markup or competitive pricing conditions

$$p_{Mt}=\frac{1+r_t}{1-\alpha}, p_{Rt}= (1+r_t), p_{It}= q_t(1+r_t^*)$$

$$p^*_{Mt}= q_t^{-1}\frac{1+r_t}{1-\alpha}, p^*_{Rt}= (1+r^*_t), p^*_{It}= (1+r^*_t).$$

The final goods firms demand schedules then yield

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}$$
$$x^*_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H^* (p^*_{jt})^{-\frac{1}{\alpha}},$$

The first-order condition for innovation at Northern intermediate goods firms, together with symmetry across firms and the equilibrium price and quantity decisions laid out above, yields the innovation optimality conditions

$$g^{\gamma-1}_{t+1}= \Omega(1+r_{t+1})^{-\frac{1}{\alpha}} \left(H + q^{\frac{1}{\alpha}}_{t+1} H^*\right),$$

which uniquely pin down the variety growth rate g_{t+1} as a function of terms of trade and interest rates. Given our characterization of g_t as a function of prices, it only remains to pin down C_t and C^*_t as a function of prices. But this is easily accomplished by noting that

$$C_t+M_{t+1}(x_{Mt+1}+x^*_{Mt+1}) + R_{t+1}x_{Rt+1}+Z_t= Y_t$$

$$Y_t= H^\alpha \left[M_t x^{1-\alpha}_{Mt} + R_t x^{1-\alpha}_{Rt} + I_t x^{1-\alpha}_{It}\right]$$

$$Z_t=\sum_{f=1}^{N} Z_{ft}=\frac{g^\gamma_{t+1}}{\gamma} A_t$$

$$C^*_t+I_{t+1}(x_{It+1}+x^*_{It+1}) + R_{t+1}x^*_{Rt+1}= Y^*_t$$

$$Y^*_t= (H^*)^\alpha \left[M_t(x^*_{Mt})^{1-\alpha} + R_t(x^*_{Rt})^{1-\alpha} + I_t(x^*_{It})^{1-\alpha}\right]$$

$$A_{t+1}= (1+g_{t+1})A_t$$

$$M_{t+1}= g_t A_t$$

$$R_{t+1}= (1-\phi_{t+1})A_t$$

$$I_{t+1}= \phi_{t+1}A_t.$$

Since all allocations in this economy are therefore a function of the $3(T-1)$ prices, we can construct the errors in $3(T-1)$ equations above given any input sequence of prices. The percentage squared errors of this system of equation are minimized using particle swarm optimization. After solving for the transition path price paths, we check to see if the cost advantage for I goods production is maintained by the South, justifying our M, R, I goods partitioning. This is equivalent to checking that, for each period

$$(1+r_t^*)q_t \leq (1+r_t).$$

In the baseline results shown in Section 5, we choose $T=7$.

Trapped Factors Transition Dynamics

The equilibrium conditions which we must solve to compute the transition dynamics for the trapped factors model are identical to those in the fully mobile economy, for period $2, ..., T-1$. There are, however, differences in the equilibrium conditions in the period of the shock. In particular, there is heterogeneity in the response of the affected and unaffected industries to the shock, and instead of solving for simply the $3(T-1)$ prices $\{q_t, r_t, r_t^*\}_{t=2}^T$ as in the fully mobile case, we must solve for these prices and the four additional variables $\{g_2^1, g_2^2, \mu^1, \mu^2\}$. These variables are patenting rates and shadow values of inputs within Northern firms in the unaffected industry (1) and the affected industry (2). Therefore, we must pin down $3(T-1)+4$ quantities, which we do with $3(T-1)+4$ equations:

$$q_1 = \left[\frac{\phi' H}{H\left[\left(\frac{n}{2}\right)(\mu^1)^{\frac{\alpha-1}{\alpha}} g_1^1 + \left(\frac{n}{2}\right)(\mu^2)^{\frac{\alpha-1}{\alpha}} g_1^2\right]}\right]^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_1}{1+r_1^*}\right)^{\frac{1-\alpha}{2-\alpha}}$$

$$q_t = \left(\frac{\phi' H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}}, 2, ..., T-1$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_{t+1}), t=1, ..., T-1$$

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} = \beta(1+r_{t+1}^*), t=1, ..., T-1$$

$$(Ng_1^1)^{\gamma-1} = \Omega(1+r_1)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H + q_1^{\frac{1}{\alpha}} H^*)$$

$$(Ng_1^2)^{\gamma-1} = \Omega(1+r_1)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_1^{\frac{1}{\alpha}} H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}} H + \frac{1}{N}\frac{g(\phi)^{\gamma}}{\gamma} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H + q(\phi)^{\frac{1}{\alpha}} H^*)$$

$$= \frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}} H + \frac{N^{\gamma-1}}{\gamma}\left(g_1^1\right)^{\gamma}$$

$$+g_1^1(1-\alpha)^{\frac{2}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H+q_1^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}H+\frac{1}{N}\frac{g(\phi)^\gamma}{\gamma}+\frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H+q(\phi)^{\frac{1}{\alpha}}H^*)$$

$$=\frac{1}{N}\chi_2(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}H+\frac{N^{\gamma-1}}{\gamma}\left(g_1^2\right)^\gamma$$

$$+g_1^2(1-\alpha)^{\frac{2}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H+q_1^{\frac{1}{\alpha}}H^*).$$

The first $3(T-1)$ equations are simply the balanced trade and Euler equations for the Northern and Southern households in periods $1,...,T-1$. The balanced trade condition must be modified in period 1 to reflect the fact that flows of M goods from North to South come from both industry 1 and industry 2, with different prices and quantities for each. The final four equations represent the innovation optimality conditions for firms in industry 1 and industry 2, as well as the trapped factors constraints for firms in each industry. The innovation optimality conditions are simply the first-order conditions of firms with respect to the mass of new varieties to be innovated in period 0 for use in period 1. Note that we are defining $\mu^1 = 1-\lambda^1$ and $\mu^2 = 1-\lambda^2$, where $m_1\lambda^1$ and $m_1\lambda^2$ are the multipliers on the trapped factors input constraints in the optimization problem for Northern intermediate goods firms in period 1. A fall in μ below 1represents a fall in the shadow value of inputs for an intermediate goods firm. Also, if M_{f1} is the number of new patents innovated by a firm in industry f in period 0 for use in period 1, we are following the conventions $g_1^f = \frac{M_{f1}}{A_0}$, and imposing the consistency condition

$$g_1=\left(\frac{N}{2}\right)(g_1^1+g_1^2).$$

The trapped factors constraints are simply the input demands for R goods production and M goods innovation and production expenditure pre-shock (left hand side) and post-shock (right hand side). The input constraints differ across industries because the R goods available in the post-shock period in industry 2 for production are reduced by the factor χ_2, where χ_2 satisfies

$$\frac{1+\chi_2}{2}=\frac{1-\phi'}{1-\phi},$$

which is the consistency condition discussed in the equilibrium definition. Also, the right-hand side on the trapped factors constraints take into account the following optimal pricing rules in the period of the shock:

$$p_{M1}^1 = \mu^1\frac{1+r_1}{1-\alpha}, p_{R1}^1 = (1+r_1),$$

$$p_{M1}^2 = \mu^2 \frac{1+r_1}{1-\alpha}, p_{R1}^2 = (1+r_1).$$

The demand conditions are identical to those laid out in the fully mobile section. Intermediate goods firm innovation costs on the right hand side of the trapped factors constraint are given by

$$Z_1^1 = \frac{N^{\gamma-1}}{\gamma} \left(g_1^1\right)^{\gamma}$$

$$Z_1^2 = \frac{N^{\gamma-1}}{\gamma} \left(g_1^2\right)^{\gamma},$$

which is a direct application of the definition of the innovation cost function. All of the other quantities needed for construction of the Euler equation errors and balanced trade conditions are identical to those in the fully mobile economy, with the exception of the resource constraints in the North and South in periods 0 and 1 which must be modified to read

$$Y_0 = C_0 + \left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^1 + x_{M1}^{*1}) + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^2 + x_{M1}^{*2}) + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_0 x_{R1}^1 + \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_0 x_{R1}^2$$

$$+ Z_1^1 + Z_1^2$$

$$Y_0^* = C_0^* + (1-\phi') A_0 x_{R1}^* + \phi' A_0 (x_{I1}^* + x_{I1})$$

$$Y_1 = H^{\alpha} \left[\left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^1)^{1-\alpha} + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^2)^{1-\alpha} + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_0 (x_{R1}^1)^{1-\alpha} + \right.$$

$$\left. \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_0 (x_{R1}^2)^{1-\alpha} + \phi' A_0 x_{I1}^{1-\alpha} \right]$$

$$Y_1^* = (H^*)^{\alpha} \left[\left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^{*1})^{1-\alpha} + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^{*2})^{1-\alpha} + (1-\phi') A_0 (x_{R1}^*)^{1-\alpha} + \phi' A_0 (x_{I1}^*)^{1-\alpha} \right].$$

After computing the transition path in the above manner, we must verify that $\mu^1, \mu^2 < 1$, justifying our imposition of the trapped factors inequality constraint as an equality constraint. We must also check the Southern cost dominance condition for Igoods in each period, i.e.

$$\min(\mu^1, \mu^2)(1+r_1) \geq q_1(1+r_1^*),$$

$$(1+r_t) \geq q_t(1+r_t^*), t = 2, .., T-1,$$

$$q_0, q_T \leq 1.$$

Welfare Calculations

We illustrate our method of computing the consumption equivalent variation by explicitly laying out the formulas used to compute the welfare gains to trade from the fully mobile trade shock. All other welfare calculations are similar.

$$W^{NS}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{NS}\right)^{1-\sigma}}{1-\sigma},W^{*NS}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*NS}\right)^{1-\sigma}}{1-\sigma}$$

$$W^{FM}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{FM}\right)^{1-\sigma}}{1-\sigma},W^{*FM}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*FM}\right)^{1-\sigma}}{1-\sigma},$$

where the consumption allocations on the fully mobile "FM" computed transition path from $0,...,T-1$ are directly computed and consumption is assumed to grow at the rate $g(\phi')$ for all economies from period T onwards. The no shock "NS" case is consumption assuming that allocations are those of the pre-shock balanced growth path with constant growth at rate $g(\phi)$. Then, we solve for x and x^*,

$$\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{NS}(1+x)\right)^{1-\sigma}}{1-\sigma}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{FM}\right)^{1-\sigma}}{1-\sigma},$$

$$\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*NS}(1+x^*)\right)^{1-\sigma}}{1-\sigma}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*FM}\right)^{1-\sigma}}{1-\sigma}.$$

The welfare numbers reported in the text are $100x$ and $100x^*$.

Price vs Variety Output Counterfactuals

This section provides explicit formulas for the price vs variety decompositions discussed in the main text. Along the fully mobile transition path in shock period 1, we have interest rates and terms of trade which determine prices and therefore intensive margins for each variety of good, say x_{I1}, x_{R1}, x_{M1}. Holding these intensive margins constant, we have that baseline Northern output in shock period 1 is given by

$$Y_1=H^{\alpha}(M_1x_{M1}^{1-\alpha}+R_1x_{R1}^{1-\alpha}+I_1x_{I1}^{1-\alpha})$$

$$M_1=A_0g_1,\ R_1=A_0(1-\phi'),\ I_1=\phi'A_0,$$

and in the no price case with R to I conversion shut down we have

$$Y_1^{noprice}=H^{\alpha}(M_1x_{M1}^{1-\alpha}+A_0(1-\phi)x_{R1}^{1-\alpha}+A_0\phi x_{I1}^{1-\alpha}),$$

with the no variety case given by

$$Y_1^{novariety}=H^{\alpha}(A_0g_0x_{M1}^{1-\alpha}+A_0(1-\phi)x_{R1}^{1-\alpha}+A_0\phi x_{I1}^{1-\alpha}).$$

Trapped factors versions reported in the text require generalization to the case of two separate industries' M goods varieties but are straightforward versions of the above. Along a balanced growth path with constant trade restriction ϕ, we have that the baseline Northern output level in a given period, with the (arbitrary) level of varieties in that particular period given by A_{ss} and considering balanced growth path intensive margins, is equal to

$$Y_{ss} = H^{\alpha}(M_{ss}x_{Mss}^{1-\alpha} + R_{ss}x_{Rss}^{1-\alpha} + I_{ss}x_{Iss}^{1-\alpha})$$

$$M_{ss} = A_{ss}g_{ss},\ R_{ss} = A_{ss}(1-\phi),\ I_1 = \phi A_{ss},$$

and the output level in the no price with no R to I conversion in that period is given by

$$Y_{ss}^{noprice} = H^{\alpha}(M_{ss}x_{Mss}^{1-\alpha} + (A_{ss} - I_{ss-1})x_{Rss}^{1-\alpha} + I_{ss-1}x_{Iss}^{1-\alpha})$$

$$I_{ss-1} = I_{ss}/(1+g_{ss}),$$

with the no variety case given by

$$Y_{ss}^{novariety} = H^{\alpha}((A_{ss} - I_{ss-1})x_{Rss}^{1-\alpha} + I_{ss-1}x_{Iss}^{1-\alpha})$$
$$= Y_{ss}^{noprice} - H^{\alpha}M_{ss}x_{Mss}^{1-\alpha}.$$

Appendix D - Semi-endogenous Growth Model

In this Appendix we consider the semi-endogenous growth model approach to show that it delivers quantitatively similar results to our fully endogenous growth model. As documented in Jones (1995a,b) the implication of a model like that considered in the main text, with "strong scale effects" implying that the long-term growth rate is dependent upon the level of human capital, is rejected by the time series evidence which documents the concurrence of rising populations and researcher numbers with constant growth rates. Jones proposes a small modification to the production function for new varieties, or alternatively, to the cost function for innovation, which implies smaller returns from the existing stock of varieties in the production of new patents. This change to the model converts the structure into a "semi-endogenous" growth model with "weak scale effects," since the long-term growth rate is now proportional to the growth rate of human capital rather than the level of human capital. Analogously, in our context with product-cycle trade, such a modification of the model leads to long-term growth rates proportional to human capital growth rates and, crucially, independent of the trade liberalization policy ϕ. As we will see, however, a reasonable calibration of a semi-endogenous growth model consistent with the data on both per-capita growth rates and population growth displays extremely long transition dynamics and considerable temporary effects on variety growth rates from trade liberalization. Therefore, the temporary growth effects of liberalization (and the permanent level effects), imply similar results for welfare regardless of whether one considers a strong or weak scale effects model. Given that the model with strong scale effects delivers closed-form expressions for the balanced growth path growth rates dependent upon the trade policy parameter ϕ, and given that the transition dynamics for the strong scale effects model are of a more reasonable length, we prefer to work with the strong scale effects model as our baseline version.

Model

We now lay out the model structure and equilibrium concept in the semi-endogenous growth framework, for the fully mobile environment only. Population and Human Capital We assume that in the North and in the South there is a continuum of identical households of measure 1, each with an expanding set of members $[0, L_t]$ and $[0, L_t^*]$, respectively. We further assume that there is an constant level of human capital per member of the population, i.e. $H_t = hL_t$ and $H_t^* = hL_t^*$, respectively. This assumption implies that preferences of the CRRA form defined over per-capita consumption or over consumption expressed relative to human capital differ only by a constant, and for convenience we express preferences as per unit of human capital.[28]

[28] Note that we omit below a term multiplying per capita preferences by the size of the population, which would be proportional to H_t^* given our assumptions. Such an assumption, as will be seen below, results in a level shift in interest rates. However, and importantly, our assumption prevents the mechanical inflation of the welfare gains from trade liberalization (relative to our baseline strong scale effects model with no population growth) simply because liberalization gains

Northern Households Given a sequence of wages w_t, firm stock prices q_{ft}, firm dividends D_{ft}, and interest rates r_t, a Northern household supplies labor inelastically and chooses consumption C_t, portfolio positions S_{ft}, and bond purchases B_{t+1} to solve the problem

$$\max_{C_t, B_{t+1}, S_{ft}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{H_t}\right)^{1-\sigma}}{1-\sigma}$$

$$C_t + B_{t+1} + \sum_{f=1}^{N} q_{ft}(S_{ft} - S_{ft-1}) \leq w_t H_t + (1 + r_t) B_t + \sum_{f=1}^{N} S_{ft} D_{ft}$$

Southern Households Given a sequence of wages w_t^*, firm stock prices q_{ft}^*, firm dividends D_{ft}^*, and interest rates r_t^*, a Southern household supplies labor inelastically and chooses consumption C_t^*, portfolio positions S_{ft}^*, and bond purchases B_{t+1}^* to solve the problem

$$\max_{C_t^*, B_{t+1}^*, S_{ft}^*} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t^*}{H_t^*}\right)^{1-\sigma}}{1-\sigma}$$

$$C_t^* + B_{t+1}^* + \sum_{f=1}^{N} q_{ft}^*(S_{ft}^* - S_{ft-1}^*) \leq w_t^* H_t^* + (1 + r_t^*) B_t^* + \sum_{f=1}^{N} S_{ft}^* D_{ft}^*$$

Northern Final Goods Firms Taking as given a sequence of wages w_t and intermediate goods prices p_{jt} for each variety $j \in [0, A_t]$ as given, perfectly competitive Northern final goods firms choose input demands H_t and x_{jt} to solve the static problem

$$\max_{H_t, x_{jt}} Y_t - \int_0^{A_t} p_{jt} x_{jt} dj - w_t H_t$$

$$\max_{H_t, x_{jt}} H_t^{\alpha} \int_0^{A_t} x_{jt}^{1-\alpha} dj - \int_0^{A_t} p_{jt} x_{jt} dj - w_t H_t$$

Southern Final Goods Firm Taking as given a sequence of wages w_t^* and intermediate goods prices p_{jt}^* for each variety $j \in [0, A_t]$as given, perfectly competitive Southern final goods firms choose input demands H_t^* and x_{jt}^* to solve the static problem

$$\max_{H_t^*, x_{jt}^*} Y_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj - w_t^* H_t^*$$

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^{\alpha} \int_0^{A_t} (x_{jt}^*)^{1-\alpha} dj - \int_0^{A_t} p_{jt}^* x_{jt}^* dj - w_t^* H_t^*$$

Northern Intermediate Goods Firms Taking as given a sequence of interest rates r_t, along with aggregate variety stocks A_t, as well as Northern and Southern final goods firms' intermediate demand schedules, each of N Northern intermediate goods firm $f$makes monopoly production x_{Mjt+1} and x_{Mjt+1}^*, perfectly competitive production x_{Rjt+1}, and

occur in the future with a larger population. In unreported results, however, we also solved an alternative model with per-capita preferences weighted by population size. Predictably, this resulted in larger welfare gains from trade liberalization.

innovation decisions M_{ft+1} to solve the following problem

$$\max_{x_{Rjt+1}, x_{Mjt+1}, M_{ft+1}} \sum_{t=0}^{\infty} m_t D_{ft},$$

$$D_{ft} + Z_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj,$$

where $\frac{m_{t+1}}{m_t} = \frac{1}{1+r_{t+1}}$ or $m_t = \Pi_{\tau=1}^{t} \frac{1}{1+r_\tau}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Northern Household's first order condition for S_{ft} and insertion of the Northern household first order condition for B_{t+1}. At all times, the innovation cost function is given by

$$Z_{ft} = \nu M_{ft+1}^{\gamma} A_t^{1-\frac{\delta}{\rho}},$$

where $\gamma = \frac{1}{\rho}$ and $\delta \in (0,1)$, and $\nu = \frac{N^{\gamma-1}}{\gamma}$ is again a scaling constant discussed in more detail below. This innovation cost function is identical to the strong scale effects innovation cost function, with the exception that $\delta < 1$ here and $\delta = 1$ in that case.

Southern Intermediate Goods Firms Taking as given a sequence of interest rates r^*_t, as well as Northern and Southern final goods firms' intermediate demand schedules, each Southern intermediate goods firm makes perfectly competitive production x_{Ijt}, x^*_{Ijt}, and x^*_{Rjt} decisions to solve the following problem

$$\max_{x_{Ijt}, x^*_{Ijt}, x^*_{Rjt}} \sum_{t=0}^{\infty} m^*_t D^*_{ft},$$

$$D^*_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj$$

where $\frac{m^*_{t+1}}{m^*_t} = \frac{1}{1+r^*_{t+1}}$ or $m^*_t = \Pi_{\tau=1}^{t} \frac{1}{1+r^*_\tau}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Southern Household's first order condition for S_{ft} and insertion of the Southern Household's first order condition for B^*_{t+1}.

Terms of Trade Notation/No Arbitrage Condition

$$p_{jt} = q_t p^*_{jt}$$

Trade Restrictions and Monopoly Structure There is one-period monopoly protection for any newly innovated M goods, trade restriction for an exogenously set proportion $1 - \phi_t$ of off-patent goods labeled R goods, and imports from South to North of the exogenously set proportion ϕ_t of off-patent goods labeled I goods.

Equilibrium Definition

- Some sequence of ϕ_t is exogenously set by the Northern government
- Northern households optimize consumption, savings, and equity purchase decisions
- Southern households optimize consumption, savings, and equity purchase decisions
- Perfectly competitive Northern final goods sector optimizes human capital and intermediate goods demand
- Perfectly competitive Southern final goods sector optimizes human capital and intermediate goods demand

- Northern intermediate goods firms optimizes M goods innovation, M goods monopoly production, and fast-copier-constrained de facto perfect competition R goods production decisions
- Southern intermediate goods firms or fast copier optimize perfectly competitive R and I goods production decisions
- Trade is balanced: $I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$
- Bond markets clear: $B_t = B^*_t = 0$
- Equity markets clear: $S_{ft} + S^*_{ft} = 1$
- Human capital market clear $H^D_t = H_t$, $(H^*)^D_t = H^*_t$
- Final goods market clears/resource constraint is satisfied in the North

$$Y_t = H_t^\alpha \int_0^{A_t} x_{jt}^{1-\alpha} dj = C_t + \int_{A_{t+1}} (x_{jt+1} + x^*_{jt+1}) dj + \sum_{f=1}^{N} Z_{ft}$$

- Final goods market clears/resource constraint is satisfied in the South

$$Y_t = H_t^\alpha \int_0^{A_t} x_{jt}^{1-\alpha} dj = C^*_t + \int_{A_{t+1}} (x_{jt+1} + x^*_{jt+1}) dj$$

- Consistency conditions hold

$$\sum_{f=1}^{N} M_{ft+1} = M_{t+1} = A_{t+1} - A_t$$

$$\phi A_t = I_t, (1-\phi) A_t = R_t$$

$$\frac{H^*_t}{H_t} = \frac{H^*_0}{H_0} = \frac{\bar{H}}{H^*}$$

- Southern cost dominance for Igoods

$$q_t(1 + r^*_t) < (1 + r_t)$$

Equilibrium Conditions for Reference

For later reference in the proof of Proposition 3, we now list the equilibrium conditions in this environment. Northern Households' (HH) First Order Conditions (FOC)

$$\beta^t H_t^{\sigma-1} C_t^{-\sigma} = \lambda_t$$
$$\lambda_t = (1 + r_{t+1})\lambda_{t+1}$$

$$\lambda_t \left(D_{ft} - q_{ft}\right) + \lambda_{t+1} q_{ft+1} = 0$$

$$\rightarrow (1 + r_{t+1}) = \frac{1}{\beta}\frac{H_{t+1}}{H_t}\left(\frac{C_{t+1}}{H_{t+1}}\frac{H_t}{C_t}\right)^{\sigma} = \frac{1}{\beta}(1+g_H)\left(\frac{c_{t+1}}{c_t}\right)^{\sigma}, \quad c_t \equiv \frac{C_t}{H_t}$$

$$\rightarrow q_{ft} = \sum_{t=0}^{\infty} m_t D_{ft}, \quad m_t \equiv \frac{\lambda_t}{\lambda_0} = \prod_{\tau=1}^{t}\frac{1}{1+r_\tau}$$

Southern Households' FOC's

$$\rightarrow (1 + r^*_{t+1}) = \frac{1}{\beta}\frac{H^*_{t+1}}{H^*_t}\left(\frac{C^*_{t+1}}{H^*_{t+1}}\frac{H^*_t}{C^*_t}\right)^{\sigma} = \frac{1}{\beta}(1+g_H)\left(\frac{c^*_{t+1}}{c^*_t}\right)^{\sigma}, \quad c^*_t \equiv \frac{C^*_t}{H^*_t}$$
$$\rightarrow q^*_{ft} = \sum_{t=0}^{\infty} m^*_t D^*_{ft}, \quad m^*_t \equiv \frac{\lambda^*_t}{\lambda^*_0} = \prod_{\tau=1}^{t}\frac{1}{1+r^*_\tau}$$

Northern Final Goods Firm FOC's

$$(1-\alpha)H_t^{\alpha} x_{jt}^{-\alpha} - p_{jt} = 0 \rightarrow x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} p_{jt}^{-\frac{1}{\alpha}} H_t$$
$$\alpha H_t^{\alpha-1} x_{jt}^{1-\alpha} - w_t = 0$$

Southern Final Goods Firm FOC's

$$(1-\alpha)(H^*_t)^{\alpha}(x^*_{jt})^{-\alpha} - p^*_{jt} = 0 \rightarrow x^*_{jt} = (1-\alpha)^{\frac{1}{\alpha}}(p^*_{jt})^{-\frac{1}{\alpha}} H^*_t$$
$$\alpha(H^*_t)^{\alpha-1}(x^*_{jt})^{1-\alpha} - w^*_t = 0$$

Northern Intermediate Goods Firm FOC's

$$\max_{x_{Mt+1}, M_{ft+1}, x_{Rt+1}} \sum_{t=0}^{\infty} m_t D_{ft}$$

$$D_{ft} = \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt})dj - Z_{ft} - \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj$$

$$-m_t\left[\frac{\partial}{\partial M_{ft+1}} Z_{ft} + x_{Mt+1} + x^*_{Mt+1}\right] + m_{t+1} p_{Mt+1}(x_{Mt+1} + x^*_{Mt+1}) = 0$$

$$p_{Mt+1} = \arg\max_p -m_t(1-\alpha)^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}}(H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1}) + m_{t+1}(1-\alpha)^{\frac{1}{\alpha}} p^{1-\frac{1}{\alpha}}(H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1})$$

$$p_{Mt+1} = \frac{m_t}{m_{t+1}}\frac{1}{1-\alpha}$$

$$-m_t + m_{t+1} p_{Rt+1} = 0$$

$$\rightarrow p_{Mt+1} = \frac{1+r_{t+1}}{1-\alpha}, \quad x_{Mt+1} = (1-\alpha)^{\frac{2}{\alpha}}(1+r_{t+1})^{-\frac{1}{\alpha}} H_{t+1}, \quad x^*_{Mt+1} = (1-\alpha)^{\frac{2}{\alpha}}(1+r_{t+1})^{-\frac{1}{\alpha}} q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1}$$

$$\rightarrow p_{Rt+1} = 1 + r_{t+1}, \quad x_{Rt+1} = (1-\alpha)^{\frac{1}{\alpha}}(1+r_{t+1})^{-\frac{1}{\alpha}} H_{t+1}$$

$$\rightarrow \frac{\partial}{\partial M_{ft+1}} Z_{ft+1} = g_{At+1}^{\gamma-1} A_t^{\frac{1-\delta}{\rho}}, \text{ imposes symmetry } g_{Aft+1} = (1/N) g_{At+1}$$

$$\rightarrow Z_t = \sum_{f=1}^{N} Z_{ft} = \frac{g_{At+1}^{\gamma} A_t^{1+\frac{1-\delta}{\rho}}}{\gamma}, \text{ imposes symmetry } g_{Aft+1} = (1/N) g_{At+1}$$

$$\rightarrow g_{At+1}^{\gamma-1} A_t^{\frac{1-\delta}{\rho}} = \Omega (1+r_{t+1})^{-\frac{1}{\alpha}} \left(H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H_{t+1}^* \right)$$

Southern Intermediate Goods Firm FOC's

$$\max \sum_{t=0}^{\infty} m_t^* D_{ft}^*,$$

$$D_{ft}^* = \int_{A_{ft}} p_{jt}(x_{jt} + x_{jt}^*) dj - \int_{A_{ft+1}} (x_{jt+1} + x_{jt+1}^*) dj$$

$$-m_t^* + m_{t+1}^* p_{Rt+1}^* = 0$$

$$-m_t^* + m_{t+1}^* p_{It+1}^* = 0$$

$$\rightarrow p_{Rt+1}^* = (1+r_{t+1}^*), \quad x_{Rt+1}^* = (1-\alpha)^{\frac{1}{\alpha}} (1+r_{t+1}^*)^{-\frac{1}{\alpha}} H_{t+1}^*$$

$$\rightarrow p_{It+1}^* = (1+r_{t+1}^*), p_{It+1} = q_{t+1} p_{It+1}^*, \quad x_{It+1}^* = (1-\alpha)^{\frac{1}{\alpha}} (1+r_{t+1}^*)^{-\frac{1}{\alpha}} H_{t+1}^*,$$

$$x_{It+1} = (1-\alpha)^{\frac{1}{\alpha}} (1+r_{t+1}^*)^{-\frac{1}{\alpha}} q_{t+1}^{-\frac{1}{\alpha}} H_{t+1}$$

Balanced Trade Condition

$$I_t p_{It} x_{It} = M_t p_{Mt} x_{Mt}^*$$

$$\phi_t A_{t-1} q_t (1+r_t^*)(1-\alpha)^{\frac{1}{\alpha}} (1+r_t^*)^{-\frac{1}{\alpha}} q_t^{-\frac{1}{\alpha}} H_t = g_{At} A_{t-1} \frac{1+r_t}{1-\alpha} (1-\alpha)^{\frac{2}{\alpha}} (1+r_t)^{-\frac{1}{\alpha}} q_t^{\frac{1}{\alpha}} H_t^*$$

$$q_t = \left(\frac{\phi_t H_t}{g_{At} H_t^*} \right)^{\frac{\alpha}{2-\alpha}} \left(\frac{1+r_t}{1+r_t^*} \right)^{\frac{1-\alpha}{2-\alpha}} \Psi, \quad \Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$$

Northern Resource Constraint

$$\begin{aligned} Y_t &= H_t^{\alpha} \left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-\alpha} + I_t x_{It}^{1-\alpha} \right] \\ &= C_t + M_{t+1} \left(x_{Mt+1} + x_{Mt+1}^* \right) + R_{t+1} x_{Rt+1} + Z_t \end{aligned}$$

Southern Resource Constraint

$$\begin{aligned} Y_t^* &= (H_t^*)^{\alpha} \left[M_t (x_{Mt}^*)^{1-\alpha} + R_t (x_{Rt}^*)^{1-\alpha} + I_t (x_{It}^*)^{1-\alpha} \right] \\ &= C_t^* + R_{t+1} x_{Rt+1}^* + I_{t+1} (x_{It+1} + x_{It+1}^*) \end{aligned}$$

Consistency Conditions and Terms of Trade Notation Convention

$$M_{t+1} = A_{t+1} - A_t, \quad R_{t+1} = (1-\phi_{t+1}) A_t, \quad I_{t+1} = \phi_{t+1} A_t$$

$$M_{t+1} = \sum_{f=1}^{N} M_{ft+1}, \quad p_{jt} = q_t p_{jt}^*$$

Southern Cost Dominance for I Goods

$$q_t (1+r_t^*) \leq (1+r_t)$$

Proposition 3 *A balanced growth path with constant ϕ exists and is unique. On this balanced growth path the growth rate g_A of varieties satisfies*

$$(1+g_A)^{\frac{1-\delta}{\rho}} = (1+g_H),$$

interest rates satisfy

$$1+r = 1+r^* = \frac{1}{\beta}(1+g_H)(1+g_A)^{\sigma},$$

and the terms of trade satisfies

$$q = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi, \Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}.$$

On this unique balanced growth path, output and consumption grow as the factor $(1+g_H)(1+g_A)$ and per capita consumption has growth rate equal to the number of varieties g_A.

Proof of BGP Formulas Assume constant growth rates of quantities and a constant ϕ. Then the HH Euler equations yield

$$1+r = \frac{1}{\beta}(1+g_H)(1+g_c)^{\sigma}$$
$$1+r^* = \frac{1}{\beta}(1+g_H)(1+g_{c^*})^{\sigma},$$

which implies that interest rates are constant. But the BT condition is then

$$q = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \left(\frac{1+r}{1+r^*}\right)^{\frac{1-\alpha}{2-\alpha}} \Psi,$$

which implies that the terms of trade are constant. But the innovation FOC is

$$g_A^{\gamma-1} A_t^{\frac{1-\delta}{\rho}} = \Omega(1+r)^{-\frac{1}{\alpha}}\left(H_{t+1} + q^{\frac{1}{\alpha}} H^*_{t+1}\right).$$
$$LHS \propto \left((1+g_A)^{\left(\frac{1-\delta}{\rho}\right)}\right)^t, \quad RHS \propto (1+g_H)^t$$

$$\rightarrow (1+g_A)^{\frac{1-\delta}{\rho}} = (1+g_H) \text{ on any BGP}$$

Now note that prices of all goods are constant because they are functions of interest and terms of trade, so the intensive demand margins are also constant multiples of human capital. In particular,

$$x_{Mt} = (1-\alpha)^{\frac{2}{\alpha}}(1+r)^{-\frac{1}{\alpha}} H_t, \quad x^*_{Mt} = (1-\alpha)^{\frac{2}{\alpha}}(1+r)^{-\frac{1}{\alpha}} q^{\frac{1}{\alpha}} H^*_t$$
$$x_{Rt} = (1-\alpha)^{\frac{1}{\alpha}}(1+r)^{-\frac{1}{\alpha}} H_t, \quad x^*_{Rt} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} H^*_t$$

$$x_{It} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} q^{-\frac{1}{\alpha}} H_t$$

$$x^*_{It} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} H^*_t$$

Note also that by the consistency conditions $M_t = g_A A_{t-1}, R_t = (1-\phi)A_{t-1}, I_t = \phi A_{t-1}$ are all constant multiples of A_t (given the fact that $A_{t-1} = \frac{1}{1+g_A}A_t$).

$$Y_t = H_t^{\alpha}\left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-\alpha} + I_t x_{It}^{1-\alpha}\right]$$
$$Y_t \propto H_t A_t \propto ((1+g_H)(1+g_A))^t$$

Now from the uses identity we also have

$$Y_t = C_t + M_{t+1}\left(x_{Mt+1} + x^*_{Mt+1}\right) + R_{t+1}x_{Rt+1} + Z_t$$

But from above

$$M_{t+1}\left(x_{Mt+1}+x^*_{Mt+1}\right)\propto H_tA_t$$

$$R_{t+1}x_{Rt+1}\propto H_tA_t$$

$$Z_t=\frac{g_A^{\gamma}}{\gamma}A_t^{1+\frac{1-\delta}{\rho}}\propto A_t^{1+\frac{1-\delta}{\rho}}\propto\left((1+g_A)^{1+\frac{1-\delta}{\rho}}\right)^t$$

But since $1+g_H=(1+g_A)^{\frac{1-\delta}{\rho}}$ on any BGP by the innovation FOC, we have

$$Z_t\propto((1+g_H)(1+g_A))^t$$

Therefore, we have

$$C_t\propto((1+g_H)(1+g_A))^t,\quad c_t\propto(1+g_A)^t$$

implying that $g_c=g_A$, so that

$$1+r=\frac{1}{\beta}(1+g_H)(1+g_A)^{\sigma}.$$

Now similar reasoning shows that

$$Y^*_t\propto H^*_tA_t,\quad C^*_t\propto H^*_tA_t,\quad c^*_t\propto A_t,$$

so that

$$1+r^*=1+r$$

$$q=\left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}}\left(\frac{1+r}{1+r^*}\right)^{\frac{1-\alpha}{2-\alpha}}\Psi=\left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi.$$

Note that this final expression implies that for sufficiently small ϕ, $q<1$, which is equivalent along the BGP to Southern cost dominance in I goods. Finally, uniqueness follows from the innovation FOC

$$g_A^{\gamma-1}A_t^{\frac{1-\delta}{\rho}}=\Omega(1+r)^{-\frac{1}{\alpha}}\left(H_{t+1}+q^{\frac{1}{\alpha}}H^*_{t+1}\right).$$

After dividing both sides by $(1+g_H)^t$, we have that

$$g_A^{\gamma-1}\propto\Omega(1+r)^{-\frac{1}{\alpha}}\left(H_1+q^{\frac{1}{\alpha}}H^*_1\right).$$

Since $\gamma>1$, the LHS is increasing in g_A. Since r is increasing in g_A and q is decreasing in g_A, there is at most one solution for g_A. Since all other prices are functions of g_A, they are unique as well. Existence is shown by noting that the increasing LHS asymptotes to ∞ as $g_A\to\infty$ and to 0 as $g_A\to 0$. The decreasing RHS asymptotes to ∞ as $g_A\to 0$ (see the formula for q) and to 0 as $g_A\to\infty$ (see the formulas for $r$and q). By the continuity and monotonicity of everything involved, as well as the intermediate value theorem, g_A exists uniquely. End of Proof

Calibration Strategy

We would like to consider, as in the fully mobile environment described above, the transition path associated with a shock from the balanced growth path associated with trade policy parameter ϕ to the balanced growth path associated with trade policy parameter ϕ'. As before, we will consider the impact of a permanent and unanticipated shock moving the policy parameter from ϕ to ϕ'. The timing conventions are identical to those discussed in the fully mobile trade shock timing section in the main text. According to the OECD National Accounts Main Aggregates dataset and Population dataset, as current in early May 2013, the average total OECD real GDP per-capita growth rate from $1984 - 2000$ is equal to approximately 2.37% per year. The average OECD population growth rates over this same period is approximately equal to 0.78% per year. Now note that the balanced growth path relationship above between g_H and g_A is a logarithmic equation whose solution yields

$$\delta = 1 - \rho \frac{\log(1 + g_H)}{\log(1 + g_A)}.$$

Above, note that g_A and g_H are 10-year versions of the annual growth rates taken from OECD data. Now, with the calibration $\rho = 0.5$ from above, we have that $\delta = 0.83$. The remaining parameters to calibrate in the model are β, σ, α, $\frac{\bar{H}^*}{H}$, H_{-1}, ϕ, and ϕ'. The values for $\alpha = 2/3$, $\sigma = 1$, $\beta = 1/1.02$, and $\frac{H_t^*}{H_t} = 2.96$ are unchanged from before. The final three parameters which must be calibrated are ϕ, ϕ', and H_1. We jointly pick these three parameters so that the following three conditions hold: $\frac{I}{Y}_{\phi,BGP} = 3.9\%$, $\frac{I}{Y}_{\phi,BGP} = 7.0\%$, and the innovation first order condition for the pre-shock ϕbalanced growth path is satisfied. The first two conditions require that the model match the non-OECD to OECD trade shares which the strong scale effects model is calibrated to match. The final condition requires that the scaling of varieties to human capital at the initial condition of the transition path is consistent with the equilibrium conditions. Given the calibration, the transition path in response to a fully mobile shock moving the economy from ϕ to ϕ' can be written as a minimization problem in r_t, r_t^*, and q_t, as in the strong scale effects case. The endpoints of each series are known, because they reflect balanced growth path values.

Results

Figure D1 plots the transition path for the semi-endogenous economy in response to the trade liberalization, for variety growth, the Southern terms of trade, and Northern and Southern per-capita output growth. In fact, the transition is not complete 25 periods. Recall that a period in this calibration is one decade, so this represents a transition path which is not complete 250 years after the initial shock. However, the broad pattern of the transition path is similar to that observed in the strong scale effects model. In particular, we have that in response to trade liberalization, the appreciation of the Southern terms of trade due to the increased flow of I goods from South to North causes an increase in the variety growth rate, as well as Northern and Southern output growth rates. Variety growth rates immediately begin to fall, however, as the gains from increased variety levels fade in the semi-endogenous innovation cost function. This process is incredibly persistent, however, because the level of δ implied by OECD evidence on per capita GDP and population growth rates is quite close to 1, yielding something quantitatively similar to the strong scale effects model. Because of consumption smoothing and the implied movements in interest rates, Northern and Southern output growth rates are smoother than variety growth, yet just as persistent. Finally, as the variety growth rate and interest rates begin to return to their normal long-run levels, the Southern terms of trade q slowly converges to its new long-run value associated with ϕ'.

Table D1: Semi-endogenous Transition Path Summary

Quantity	Value
max g_{At}	2.8%
(max g_{At})$-g_A$	0.45%
Half Life	16 periods
r	5.2%
$q(\phi)$	0.46
$q(\phi')$	0.68
$\frac{I}{Y}_{\phi}$	3.9%
$\frac{I}{Y}_{\phi'}$	7.0%
ΔW	16.5%
ΔW^*	15.4%

Note: The table above displays a summary of the quantitative exercise performed for the semi-endogenous model given a calibrated trade liberalization. The long-run annualized value of the interest rate is given as r, and all other quantities are computed from a transition path in response to an unanticipated, permanent movement of trade policy $\phi$to $\phi' > \phi$, where $\phi$and $\phi'$are chosen to match the movement in low-cost imports to OECD GDP observed in the data from 1997-2006 and also displayed in the table. The pre- and post-shock Southern terms of trade $q(\phi)$and $q(\phi')$vary permanently with the trade policy parameter and reflect the balanced growth path for the indicated policy. The maximum level of variety growth max $g_{At}$and the maximum difference in variety growth from its long-run level over the transition path are displayed in the first two rows, while the half life of the shock to variety growth induced by trade liberalization is indicated in the third row. The model calibration of a period is one decade. $\Delta W$and ΔW^*refer to the permanent consumption equivalent of trade liberalization for a Northern and Southern household, respectively. In particular, this percentage is the permanent fraction by which consumption for a household must increase in each period without the trade shock to make the household indifferent to the allocation with trade liberalization.

More precisely, in Table $D1$ we present the detailed statistics associated with trade liberalization in the semi-endogenous model. In particular, note that the half-life of the shock to the variety growth rate is 16 periods, or 160 years. Also, note that the welfare gains to the North and to the South from liberalization, 16.5% and 15.4%, which are permanent consumption equivalent welfare gains defined analogously to before, are qualitatively similar to those obtained from the strong scale effects model.

Appendix E - R&D Cost Externalities within Strong Scale Effects Model

As noted in the main text, to allow for the problem that firms face in coordinating search and innovation in larger teams, we allow for a form of diminishing marginal productivity for the inputs to innovation in any given period. This diminishing marginal productivity can be internal in the sense that it depends only on the inputs devoted to innovation within the firm, or it could be external in the sense that it depends on total inputs devoted to innovation in the economy. We start first with the fully internal case, which is our benchmark structure considered in the main paper. In this case, the number of new designs at firm $f$is a function of innovation expenditures Z_{ft} within firm f:

$$M_{ft+1} = (Z_{ft})^{\rho} A_t^{1-\rho},$$

where $0 < \rho < 1$.This yields an internal R&D cost function given by

$$Z_{ft} = IC(M_{ft+1}^{\gamma}, A_t) = M_{ft+1}^{\gamma} A_t^{1-\gamma},$$

where $\gamma = \frac{1}{\rho} > 1$ and the function name IC is a mnemonic for Internal Costs. The other extreme, which is the extension we consider in this section, would be to assume that the costs of innovation for any one firm depend on the total amount of innovation that is taking place in the economy because independent firms could develop redundant designs. In this case, with fully external increasing costs, the aggregate production function for innovation is given by

$$M_{t+1} = (Z_t)^{\rho} A_t^{1-\rho},$$

where Z_t is the aggregate quantity of final good devoted to innovation. The corresponding aggregate cost function is

$$Z = M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

In this case, the cost per new patent to an individual firm would be the average economy-wide cost of innovation

$$Z_{ft} = EC(M_{ft+1}, M_{t+1}, A_t) = \frac{M_{ft+1}}{M_{t+1}} M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

where EC is a mnemonic for external costs. To allow for intermediate degrees of internal and external costs of innovation, we nest these two versions in a cost function for firm $f$of the form

$$Z_{ft} = \nu \left(IC(\bullet)\right)^{\eta} \left(EC(\bullet)\right)^{1-\eta},$$

where $0 \leq \eta \leq 1$ and the inputs for the functions $IC(\bullet)$ and $EC(\bullet)$ are as given above. As η increases, the cost function exhibits a steeper marginal cost curve within each firm, with less redundancy across firms and hence weaker innovation externalities. The fully internal and fully external innovation cost benchmarks are the cases of $\eta = 1$ and $\eta = 0$, respectively. The introduction of η requires a slight change in the scaling constant $\nu$to deliver invariance of balanced growth path growth rates to N, η, ρ. However, the equilibrium definition and structure is identical to that considered above, except for the obvious modifications to the innovation first-order conditions and resource constraints. For the fully mobile environment, the symmetry across firms causes invariance of the aggregate allocation to the level of η. Only the trapped factors transition dynamics are modified. For completeness, we reproduce below the modified system of equations solved numerically to compute the transition path in the trapped factors case with an arbitrary level of η. These equations are the direct analogy of those in Appendix C above.

$$q_2 = \left[\frac{\phi' H}{H \left[\left(\frac{n}{2}\right) (\mu^1)^{\frac{\alpha-1}{\alpha}} g_2^1 + \left(\frac{n}{2}\right) (\mu^2)^{\frac{\alpha-1}{\alpha}} g_2^2 \right]} \right]^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_2}{1+r_2^*} \right)^{\frac{1-\alpha}{2-\alpha}}$$

$$q_t = \left(\frac{\phi' H}{g_t H^*} \right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^*} \right)^{\frac{1-\alpha}{2-\alpha}}, 3, ..., T$$

$$\left(\frac{C_{t+1}}{C_t} \right)^{\sigma} = \beta(1 + r_{t+1}), t = 2, ..., T$$

$$\left(\frac{C_{t+1}^*}{C_t^*} \right)^{\sigma} = \beta(1 + r_{t+1}^*), t = 2, ..., T$$

$$(Ng_2^1)^{\eta(\gamma-1)}(g_2)^{(\gamma-1)(1-\eta)} = \Omega(1+r_2)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H+q_2^{\frac{1}{\alpha}}H^*)$$

$$(Ng_2^2)^{\eta(\gamma-1)}(g_2)^{(\gamma-1)(1-\eta)} = \Omega(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H+q_2^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}H + \frac{1}{N}\frac{g(\phi)^\gamma}{\eta(\gamma-1)+1} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H+q(\phi)^{\frac{1}{\alpha}}H^*)$$

$$= \frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^1\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+ g_2^1(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H+q_2^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}H + \frac{1}{N}\frac{g(\phi)^\gamma}{\eta(\gamma-1)+1} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H+q(\phi)^{\frac{1}{\alpha}}H^*)$$

$$= \frac{1}{N}\chi_2(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^2\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+ g_2^2(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H+q_2^{\frac{1}{\alpha}}H^*).$$

$$g_2 = \binom{N}{2}(g_2^1 + g_2^2).$$

$$\frac{1+\chi_2}{2} = \frac{1-\phi'}{1-\phi},$$

$$p_{M2}^1 = \mu^1\frac{1+r_2}{1-\alpha}, p_{R2}^1 = (1+r_2),$$

$$p_{M2}^2 = \mu^2\frac{1+r_2}{1-\alpha}, p_{R2}^2 = (1+r_2).$$

$$Z_2^1 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^1\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$Z_2^2 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^2\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)},$$

$$Y_1 = C_1 + \binom{N}{2}g_2^1A_1(x_{M2}^1 + x_{M2}^{*1}) + \binom{N}{2}g_2^2A_1(x_{M2}^2 + x_{M2}^{*2}) + \binom{N}{2}\frac{1-\phi}{2}A_1x_{R2}^1 + \binom{N}{2}\frac{(1-\phi)\chi_2}{2}A_1x_R^2$$

$$+ Z_2^1 + Z_2^2$$

$$Y_1^* = C_1^* + (1-\phi')A_1x_{R2}^* + \phi'A_1(x_{I2}^* + x_{I2})$$

$$Y_2 = H^\alpha\left[\binom{N}{2}g_2^1A_1(x_{M2}^1)^{1-\alpha} + \binom{N}{2}g_2^2A_1(x_{M2}^2)^{1-\alpha} + \binom{N}{2}\frac{1-\phi}{2}A_1(x_{R2}^1)^{1-\alpha}\right.$$

$$\left. + \binom{N}{2}\frac{(1-\phi)\chi_2}{2}A_1(x_{R2}^2)^{1-\alpha} + \phi'A_1x_{I2}^{1-\alpha}\right]$$

$$Y_2^* = (H^*)^\alpha \left[\left(\frac{N}{2}\right) g_2^1 A_1 (x_{M2}^{*1})^{1-\alpha} + \left(\frac{N}{2}\right) g_2^2 A_1 (x_{M2}^{*2})^{1-\alpha} + (1-\phi') A_1 (x_{R2}^*)^{1-\alpha} + \phi' A_1 (x_{I2}^*)^{1-\alpha} \right].$$

$$\min(\mu^1, \mu^2)(1+r_2) \geq q_2(1+r_2^*),$$

$$(1+r_t) \geq q_t(1+r_t^*), t = 3, .., T,$$

$$q_1, q_{T+1} \leq 1.$$

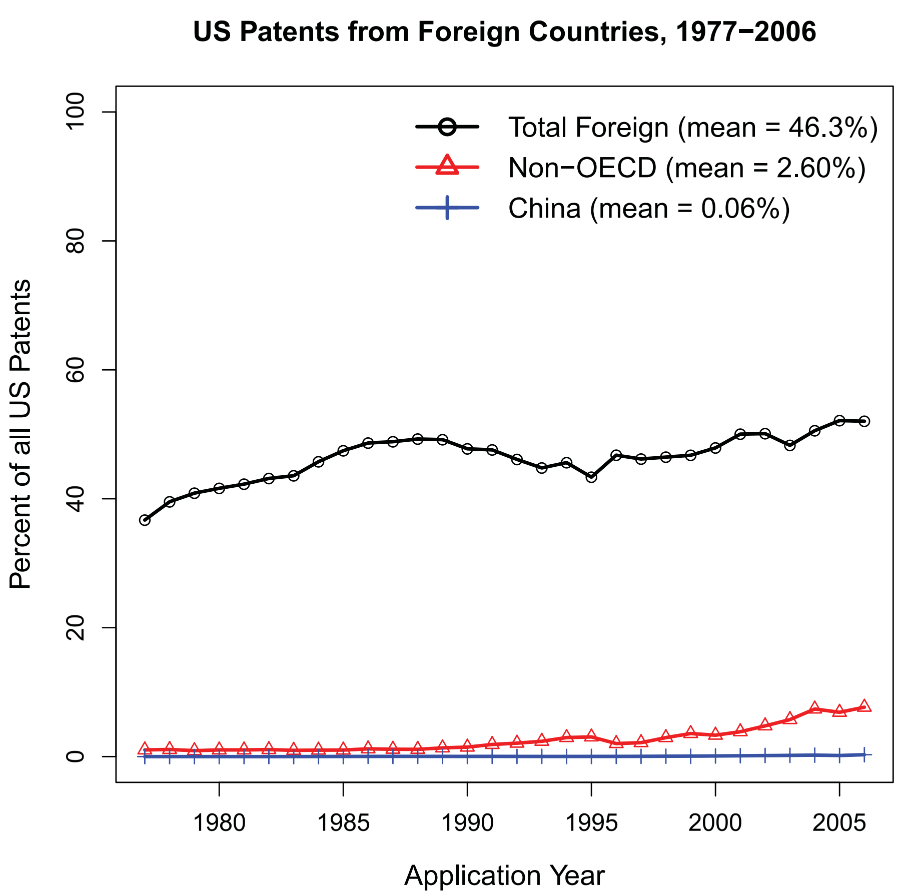


Figure B1: Non-OECD Patent Ratios are Small

Note: Patent fractions are computed from the NBER patent database, accessible via Brownyn Hall's website. Patents granted to multiple assignees are counted only once. The classification of patents by assignee to the required OECD, non-OECD, and Chinese categories is done by the citizenship of the first assignee, and a given country's OECD member status as of the application year. Each series is normalized by the total number of granted US Patent and Trademark Office applications in the same year. The reported means are computed over the full range 1977-2006.

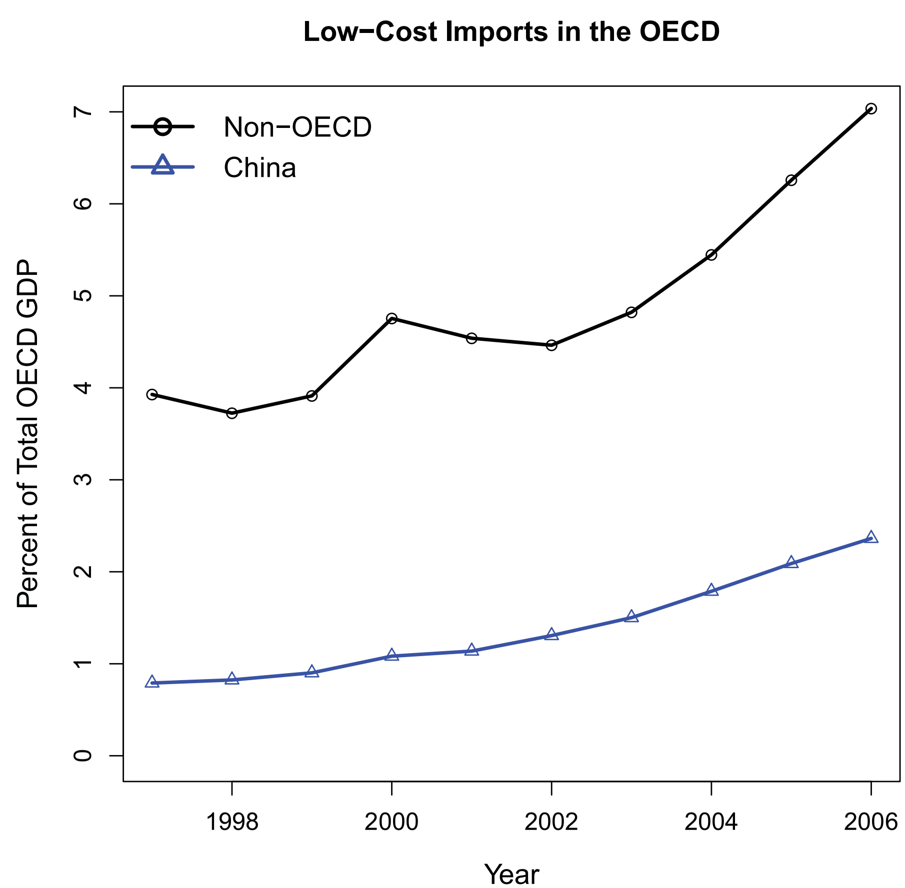


Figure B2: Import Ratios are Increasing

Note: Non-OECD and Chinese imports into OECD countries are from the OECD-STAN database as available in April 2013. Chinese import data is directly available, and non-OECD imports are imputed as the difference between world imports and imports from other OECD members in a given year. The normalizing GDP measure for the OECD is computed from the Penn World Tables version 7.1 and equals the sum of GDP for all OECD members in a given year. The Chinese imports to OECD GDP ratio in 1997 is 0.79% and in 2006 is 2.4%. The total non-OECD imports to OECD GDP ratio in 1997 is 3.9% and in 2006 is 7.0%.

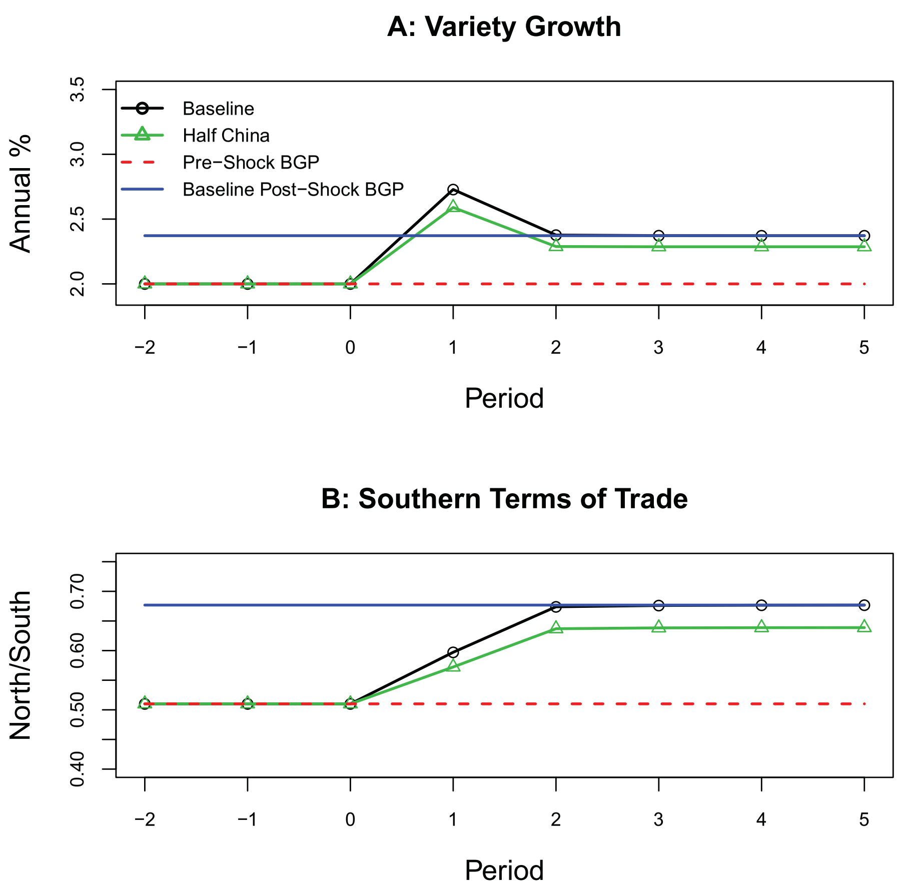


Figure B3: Trade Liberalization with Half of Chinese Import Growth

Note: The figure displays the transition path in response to trade liberalization in two scenarios. The first transition path, in solid black, "Baseline," replicates the trapped factors transition path displayed in Figure 4 above. A permanent and unanticipated trade liberalization from ϕ to $\phi' > \phi$ is announced in period 0 to become effective in period 1. The second transition path in green with triangle symbols, "Half China," plots the trapped factors transition path, starting with the same initial conditions as "Baseline" but instead considering a counterfactual increase of ϕ to a level between ϕ and ϕ' which matches post-liberalization imports to GDP ratios assuming that half the growth in Chinese imports into the OECD occurs through policy substitution to non-China, non-OECD countries. The upper horizontal solid blue line is the post-shock balanced growth path, and the lower horizontal dashed red line is the pre-shock balanced growth path.

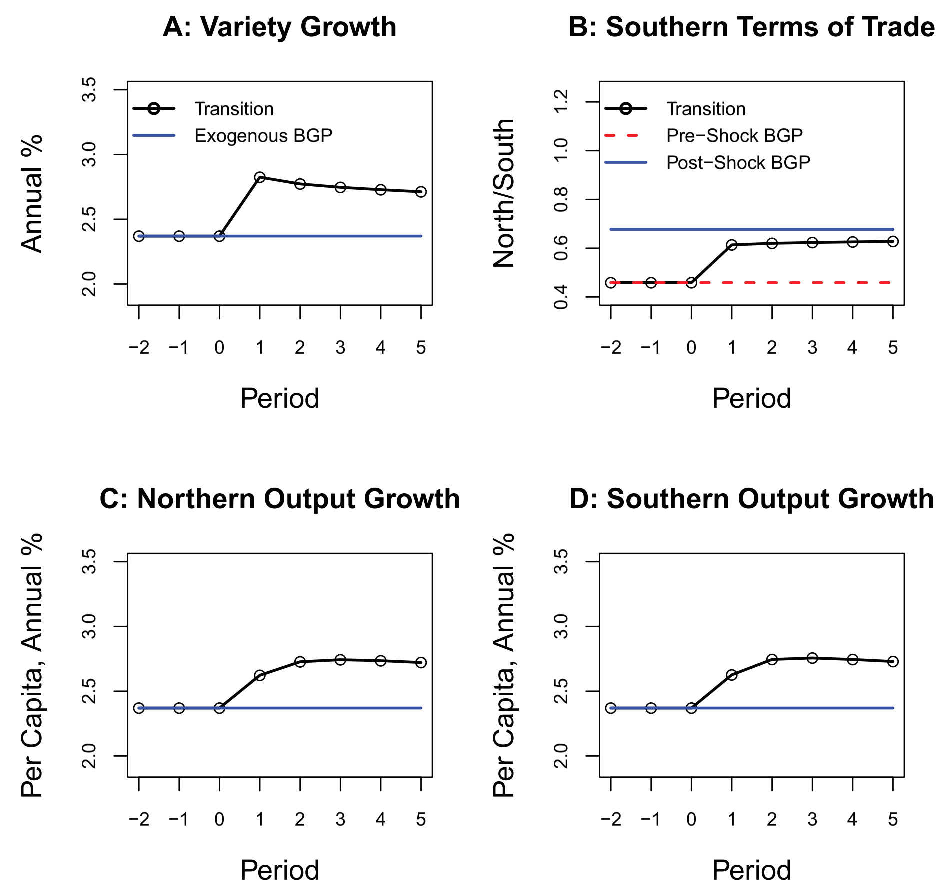
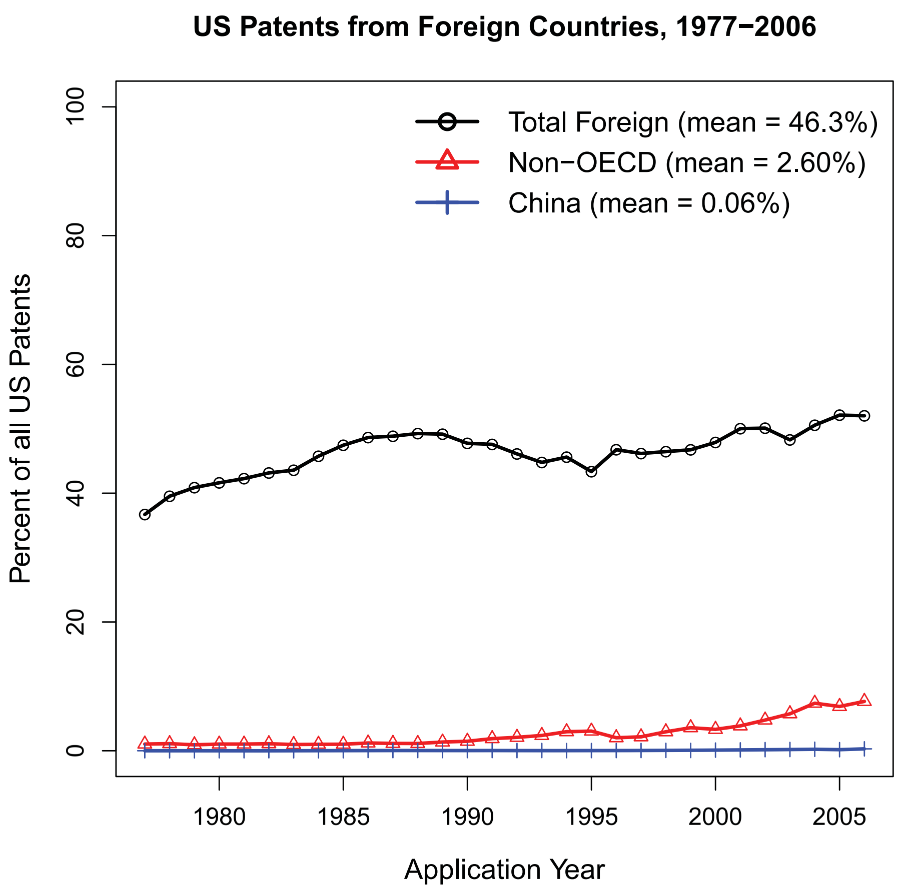


Figure D1: Semi-endogenous Growth Model Trade Liberalization

Note: The figure displays the fully mobile transition path in the semiendogenous growth model in response to a permanent, unanticipated trade liberalization from policy parameter ϕ to $\phi' > \phi$, which is announced in period 0 to become effective in period 1. Intermediate goods firms may respond to the information about trade liberalization without short-term adjustment costs. The solid black line is the transition path, the upper horizontal solid blue line is the post-shock balanced growth path, and the lower horizontal dashed red line is the pre-shock balanced growth path. Note that since the semiendogenous growth model's value for variety growth and output growth in the long run does not vary with trade policy, there is only one balanced growth marker for these series.



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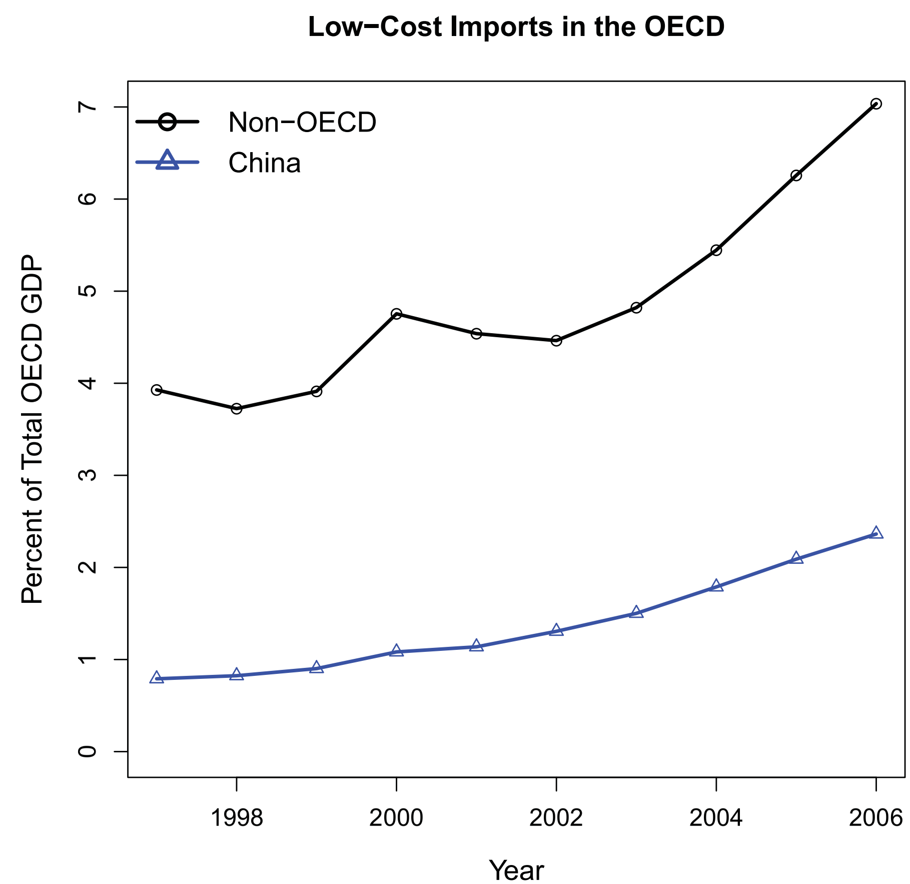

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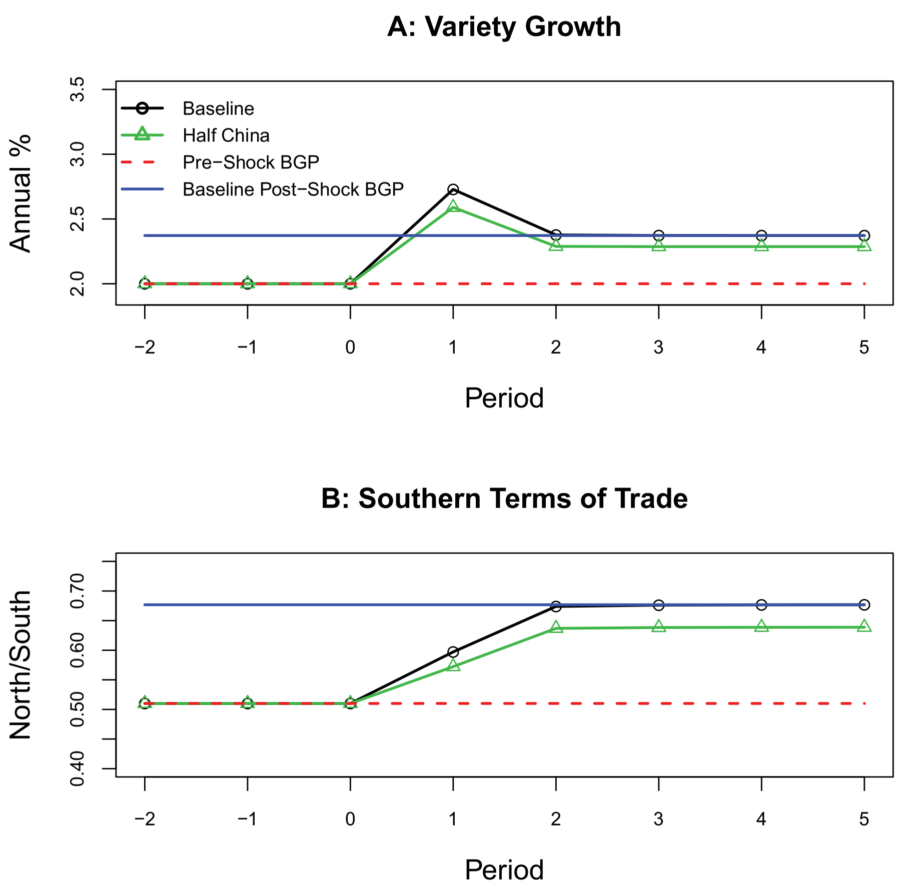

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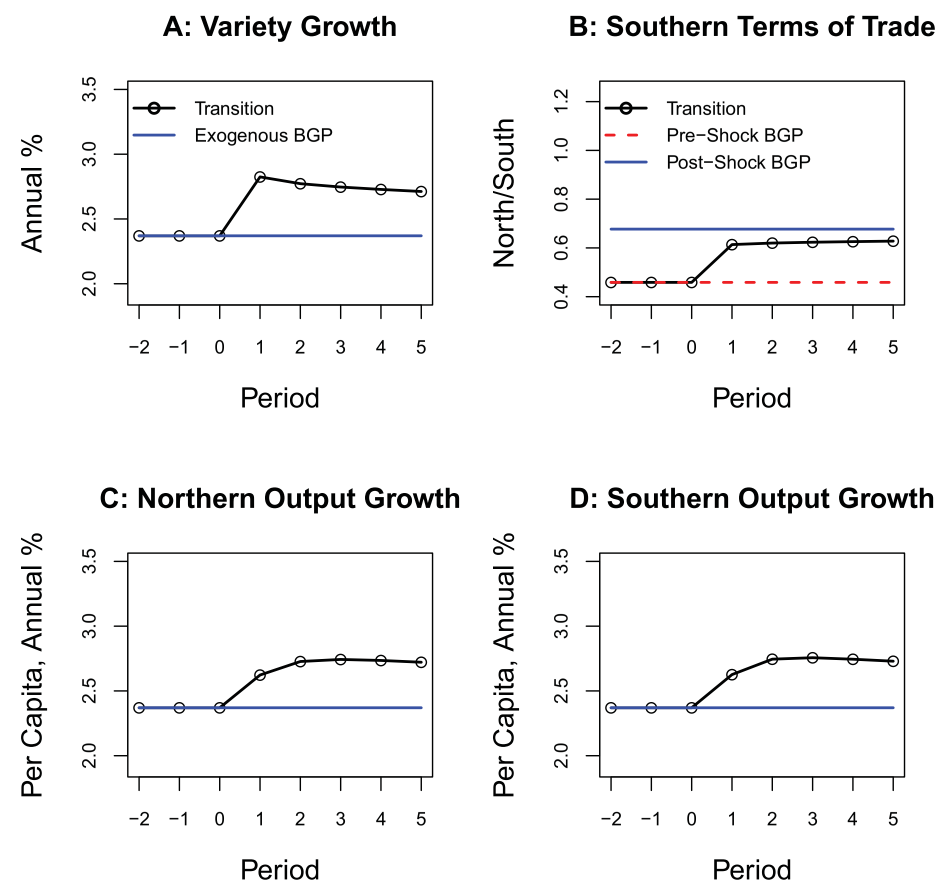

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